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Scientific philosophy in the Lvov-Warsaw School
Pragmatic rationalism as its mainstream trend

From here on, the term "School" is an abbreviated form of "the Lvov-Warsaw School". The term "science" is used here in the sense of German "Wissenschaft" to cover not only empirical but also mathematical disciplines.

§1. The main features of pragmatic rationalism

§1.1. It is the contention of this essay that the mainstream of the School's scientific and anti-irrationalist philosophy fits into the frame of *pragmatic rationalism*. Appearing also under the name of *pragmatic Platonism*, it proves represented by the host of leading logicians of the 20-th century: Russell, Gödel, Quine, Putnam. Kreisel, Chaitin. Each of them develops a version which differs each from other ones at some points. After examining these versions, one finds the Gödelian as most relevant to a comparative discussion concerning the rationalistic mainstream of the School. Therefore so much attention is paid here to Gödel's approach, discussed in Sections §2 and §3.

When speaking of the School's mainstream, I mean ideas expressed in documents which can be called the School's manifestos. One given by Kazimierz Twardowski, the other by Kazimierz Ajdukiewicz, entitled *Der logistische Antiirrationalismus in Polen*. The former is found in Twardowski's [1929] programmatic speech on the occasion of an anniversary of the Polish Philosophical Society (1929). [1]

Ajdukiewicz [1935] presented the School's ideas and achievements at an international conference. The paper's title refers to rationalism in a slightly implicit way, using the term "anti-irrationalism" – to denote the scientific – hence rationalist – philosophy in Poland. "Scientific", because that manifesto, as well as that by Twardowski, emphasizes the urgent need of interactions between philosophy and sciences. Moreover, Ajdukiewicz, with adding the attributive "logistic" to "anti-irrationalism" attributes the distinguihed role to mathematical logic (for citations see §4.1).

A word should be said about the presence of an approach competitive to the said rationalist mainstream. It is radical and fundamentalist version of empiricism represented in the School by Tadeusz Kotarbiński. Kotarbiński, in principle, favoured the idea of collaboration with sciences, but except for such cases in which his reistic philosophy entered a conflict with scientific achievements, e.g. the electromagnetic field theory of light. He rejected all field theories as devoid of any sense. For, according to the reistic ontology, there exist solids alone, while waves are no solids. The reistic attitude to sciences is discussed in a more detail in §4.2.

As to Kotarbiński's support for the philosophical use of mathematical logic, it is doubtful as well. In his textbooks, without any reservations he accepts propositional logic. However, instead of the classical predicate logic he presents in the textbooks either Leśniewski's calculus of names or the traditional syllogistics. The latter was regarded by him as a forerunner of modern calculus of names, and aimed at beginners as being more accessible. The first-order logic was suspected for him as too near to Platonism, while higher-order logics decidedly was banned for being overtly platonic.

With such an attitude, Kotarbiński could not take advantage of the enormous philosophical impact of predicate logic which is most significant in the case of higher-order logics. Such benefits of mathematical logic were available to those in the School community who followed Ajdukiewicz's rationalistic manifesto, but not those who sticked to Kotarbiński's radical nominalism and empiricism. So he proves to be less representative to the mainstream of the School. Nevertheless he should be appreciated as the main opponent of pragmatic rationalism, and so its partner in a fruitful controversy.

§1.2. There are significant differences between two varieties of rationalism. One of them which deserves to be called *classical* includes Plato, and the great classics of rationalism of the 17th century, especially Descartes and Leibniz. The other one – here called *modern* – starts from Peirce (1839-1914) and involves Frege, Gödel, and other founding fathers of mathematical logic. They differ not only with chronological setting but also with their content. The classical rationalism is *fundamentalist* while the modern one – *pragmatic*.

Let us consider this difference against the background of what is common to classical and modern approaches. This common feature is most conspicuous in the case of mathematics. Rationalism of either kind claims that mathematical propositions are meaningful and have a truth value, that is, are either true or false. There are opponents of rationalism, also in the School, who deny the meaningfulness of mathematical formulas, unless they get reduced to some statements about solids (cp. 1.1 above).

Other opponents, arguing from the point of empiricism, like those in the Vienna Circle, maintained that mathematical formulas had no cognitive content, hence no truth value. They are merely conventional rules of syntax needed to guide inferences in empirical sciences in which laws of nature are logically derived from observation statements.

[1] More on the content of Twardowski's text – in the sequel, §4.3.

When following the idea that there are true mathematical formulas, in the classical meaning of "truth" as defined by Tarski [1933], we are bound to agreee that there exist entities which these formulas speak of. Then there must exist mathematical entities. There are at least two features to distinguish them from physical and from mental objects. They exist neither in space nor in time, and they do not possess any causal power to directly influence physical processes in the world.

It is usual to name them *abstract objects*. This is the name I am to use in the sequel. There are good reasons to justify such naming, but accounting for them would need a separate sophisticated discussion [2]

Thus, to sum up, there are two points at which the two branches of rationalism, classical and modern, meet with each other. These are: the existence of abstract entities (ontological point), and their being perceived with insights of reason (epistemological point). The basic difference between the classical and the modern version is primarily epistemological (concerning the nature of intellectual insights), and just secondarily, as an corollary, it may have an ontological significance. Epistemology of classical rationalism, from Plato to Descartes, Leibniz, etc., is *fundamentalist*, while the modern one is *pragmatic*.

Let it be noted first that fundamentalism is characteristic of various epistemological views, not merely classical rationalism. Fundamentalists of all options believe that there exist unshakable foundations of knowledge, not being liable to any revision, even in a most distant future. According to rationalism, such firm foundations of knowledge are inherent in the truths disclosed by insights of reason. On the other hand, the empiricist fundamentalism maintains that such unshakable foundations are found in sense-experiences alone.

§1.3. The classical rationalism is fundamentalist on account of its claiming that the insights of reason (1) enjoy infallibility, and (2) play tho role of the firmest basis of the whole knowledge. They are always (3) given in a direct intellectual perception, never being a result of guessing, conjecturing or imagining. Therefore they (4) are safeguarded against any revisions or corrections. Therefore, too, (5) there is no need of their confronting with empirical facts.

The modern rationalism is pragmatic due to several features, each of them contrary to its counterpart in the above listing. Let the opposites be marked with asterisk. These are as follows

As for (1*) and (2*), intellectual insights are not infallible. This is to mean, they share *fallibility* with sensory perceptions. However, this does not mean the lack of credibility. There happen errors, but these are not more dramatic than, for instance, sense-illusions. This does not diminish their basic and enormous role in winning the knowledge about the world. Both in the sensory perception and in intellectual perception failures can be eliminated with taking into account a broader context of knowledge.

As for (*3 and (4*), according to pragmatic rationalism, insights into the realm of abstract objects are not rarely products, so to speak, of creative intellectual imagination. This is a power characteristic of reason alone, essentially different from sensory imagination like that of painters or musicians, or else fiction writers. It happens to rightly be called *cognitive imagination*, that is, one conceptually entertaining possibilities. [3]

(5*) Since intellectual insights are not privileged with infallibility, they ought to be checked like hypotheses in empirical science. It is no accident that some mathematical statements in modern mathematics are named with terms like "Goldbach conjecture", "continuum hypothesis", etc. Their authors had enough cognitive imagination to arrive at bold insights, but the current mathematics does not possess sufficient means to decide their truths. Before this is done they remain only tentative.

§2. Gödel's pragmatic rationalism based on his platonism and metamathematical research

MOTTO. *It is just as much objective fact that the flower has five petals as that its colour is red.*

§2.1. This short and simple sentence by Gödel, taken as the motto of the present Section, belongs to the most important statements that have ever been uttered in the history of philosophy. [4]

This statement throws a bridge across two ways of seeing the world, which for ages were regarded as denying each other: rationalism versus empiricism. Their reconciliation consists in getting aware that the same four letters "five" in one context may refer to highly abstract entity, a cardinal number, and in another context – to the structure of a concrete object, as a flower, or a human palm in its part consisting of fingers. It is the awareness to make it possible to reconcile rationalism with empiricism.

Five flakes of a flower and five fingers of a palm, in spite of all the differences, have the same structural property, that of being-five. This is a visual feature, as accessible to our eyes as petal's property of a being-red, or finger's property of being oblong. Thus colour, shape and, visual multitude belong to the same category of properties: physical and falling under senses.

Starting from such a physical and sensory level, we are able to make the first step of abstraction: to *disregard* the differences of colour and shape of five petals

[2] A duly sophisticated account can be found in the article "Abstract Objects" by Gideon Rosen [2012] in http://plato.stanford.edu/entries/abstract-objects/

[3] A penetrative discussion of the issue is found in the article by Tamar Gendler [2011] "Imagination" in http://plato.stanford.edu/entries/imagination/.

[4] See Wang [1996, loc. 3346 in Kindle format].

and five fingers, and focus attention of the property of being-five in both cases. Hence the property called *cardinality*. In the idiom of set theory one says that two sets have the same cardinality if their elements can be put in one-to-one correspondence.

Thus from the visual perception of five-elements structure we come to a more general concept of **five** (let concepts be distinguished by boldface). The term "disregard" and "focus on" hint at the very nature of *abstraction*: it consists in disregarding some features to concentrate on other ones.

Let us consider another case, also familiar in our everyday vernacular speech – the concept of **pair**. The pair of shoes and the pair of sockets differ not only with shapes but also with the fact that the pair of shoes is in a way ordered, that is, the left element cannot be replaced by the right and vice versa, while in the pair of sockets this is possible; the latter pair is not ordered.

Now we realize that what is essential for cardinality is not only to disregard such properties of elements as colour, shape, etc., but also a property of the set (here a pair) in question. Namely, its being ordered in such or another way, or not being ordered in any way. This is our second step of abstraction in the route from sense-experiences up to high regions of arithmetic and set theory.

The next step of abstraction consists in realizing that the concepts **five** and **two** (i.e., pair), and other like those, though they denote different multitudes of elements, do have something in common: each of them refers to a set. Thus they share the attribute of being a set, and this implies the existence of sets as a category of entities.

Let the third step considered be such a generalization of the concept **pair** that it embraces abstract entities on equal footing with concrete ones. This is done in the theory of sets with the axiom of pairing which introduces the concept of **pair of sets**. It runs as follows (let the letters *a, b, c, d* represent sets).

$$\forall_a \forall_b (a \neq b \Rightarrow \exists_c \forall_d (d \in c \Leftrightarrow (d = a \vee d = b)))$$

This means: for any two different sets (*a, b*), there is a set (*c*) such that for any set (*d*) it belongs to *c* if, and only if, it is identical either with *a* or with *b*. The does not distinguish whether the sets under consideration are concrete, as a pair of shoes, or abstract, as the sets of odd and of even numbers; in this sense the axiom leads to a higher level of generalization.

At the same time, the abstraction ramifies in another way. Now the concept **pair** comprises both concrete entities, as pair of sockets, and abstract ones, as a set of two sets. At this point our discourse gets distanced from the lowest level, that of visual data. Nevertheless, there remains a chain of intermediate links to connect so distant ends. This journey of mind we owe to successive acts of abstraction each of them being an insight, first, into some physical multitudes, and then into the ever more abstract multitudes.

Thus justice is done both to empiricism for its acknowledging the empirical basis of abstraction, and also to rationalism which appreciates the role of abstraction acts as a kind of intellectual insights.

§2.2. A next problem to be handled is that of the reliability of ideas being successively obtained at ever higher levels of abstraction. Have they the same reliability at each level, or does there exist a differentiation? A thought-provoking hint concerning this question is found in the following remark by Gödel.

> Strictly speaking, we only have clear propositions about physically given sets and then only about simple examples of them. If you give up idealization, mathematics disappears. — Wang [1996, Sec. 7.1.10, loc. 3419 in Kindle format].

Gödel's phrase "physically given sets" may astonish those who are accustomed to the view that sets have to always be typically abstract objects. I had met quite a number of such people, therefore I devote so much care here (§2.1 above) to explain Gödel's thought that the set of five petals may be physically given, and perceived with eyes. And this example is really very simple, as noticed by Gödel. A sense-experience after several steps of abstraction gets transformed into the abstract notion of set.

Such an overturn inside the rationalistic epistemology is imminent in the saying that clear propositions about sets are available only in the case of simple physically given sets. This implies that there is less clarity in propositions concerning abstract sets. Moreover, we learn from the context of these Gödel's considerations that the higher is the level of abstraction, the lower becomes the level of clarity and certainty in what is asserted of sets.

His point is nicely exemplified by the rise of set-theoretical antinomies with such uppermost degree of abstraction as in the notions of **the set of all sets** and **the set of all sets that are not members of themselves**. At that level our insights into the realm of sets so much lack clarity that they become misleading and inconsistent.

This is why such highly abstract ideas ought to be confronted with those at the lower levels, nearer to physical and empirical reality. So looks the Gödelian project of reconciling two claims which seemed incurably opposite: (i) the rationalist trust in the force of intellectual insights which lead our minds towards ever higher degrees of abstraction, and (ii) the empiricist caution to remain as close as possible to the concreteness of physical reality, most accessible to our senses.

§2.3. This complex of Gödelian views can be summed up in the following points.

(A) Insights of reason (A1) supply our knowledge with new concepts involved in new axioms; (A2) they postulate the existence of entities which the new ideas refer to. **(B)** The clarity and the truth of so introduced assertions are not granted by the very attribute of intuitiveness. **(C)** They remain in the moment just tentative, and should be checked by their consequences as valid either in direct sense-experience or in well-

established theories, empirical or deductive; then they obtain a desired clarity, and become closer to certainty.

Item A discloses Gödel's Platonic attitude both in epistemological (A1) and in ontological (A2) respect.

However, as seen in B, he does not share the point maintained by Plato himself, and by the classical rationalists of the 17-th century, that the assertions due to insights enjoy the privilege of full clarity and infallibility. As stated in C, they are more like conjectures considered in empirical sciences; their consequences may consist also in practical applications, e.g. in technology.

Thus B opposes the classical fundamentalist rationalism, while C introduces a genuinely pragmatic feature, to wit the fallibility of scientific assertions. The uncertainty has varying degrees; it can be diminished with successfully passed checks to result in a higher degree of confirmation.

> Here a historical comment will be in order. The novelty of pragmatic approach in the history of rationalism can be emphasized on the example of Plato's view that the insight into ideas is direct and infallible, even more certain than sensory perceptions, as these happen to be dim and vague, even illusory. On the contrary, the perceiving of ideas which human souls enjoy in the phase of pre-existence, in the realm of ideal objects, is perfectly clear, and privileged with infallibility. Classical rationalists of the 17-th century did not share Plato's vision of spiritual preexistence, but believed that intellectual insights must enjoy the same cognitive value as those which Plato spoke of. And owing to this merit they are the sole candidates to serve as solid and definite foundations of knowledge.

The evolution of science since the 17-th century has changed this landscape. As for empirical sciences, there appeared the awareness of fallibility of theories – the idea firmly promoted by such thinkers, as Peirce, Popper etc. As for mathematics, there were at least two factors which undermined the fundamentalist belief in the infallibility of mathematics: the rise of set-theoretical antinomies, and Gödel's proof of the incompleteness of number theory; the latter accompanied by the proof that its consistency cannot be demonstrated without resorting to a theory less reliable than the number theory itself.

Application is a relationship reverse, in a way, to confirmation, the latter belonging to key concepts of the methodology of science. When saying that, e.g., the theory of gravitation has *applications* concerning the movements of celestial bodies, we mean the following: the observation of these movements as conforming to the prediction of the theory, increases the degree of its *confirmation*.

The theory of gravitation, in turn, can serve as an instance of applications of arithmetic, namely the functions of multiplication, division and exponentiation as occurring in Newton's formula. Concepts which refer to these entities are applied in computing gravitation, and the successes of this computing, whenever Newton's law is applied, confirms the consistency of arithmetic.

The role of the next link (being "above" arithmetic, while physics, astronomy, etc. are "below") in this chain of applications and confirmations is played by set theory. Again, let me quote a significant saying of Gödel.

> If set theory is inconsistent, then elementary number theory is already inconsistent. — [Wang 1996, Sec. 7.1.8, loc. 3400 in Kindle format.]

In other words, if arithmetic is consistent, then set theory is consistent. As stated above, if physics is consistent, then arithmetic is consistent. Hence: if physics is consistent, then set theory is consistent. As to the consistency of a physical theory, it is always a conjecture which is to be tested with observations which result in suitable measurements.

Each measurement is rendered as a finite string of digits which are read off from indications of scientific instruments. Such digits have to refer to rather small numbers of which Gödel says in a quotation given above in §2.2. Let me recall it:

> Strictly speaking, we only have clear propositions about physically *given sets* and then only about *simple* examples of them.

Note that the sequences of digits as displayed by an instrument are physically given sets, and are simple – as containing not too long string of digits; thus it can be accessible to human eyes.

Let us trace the path back: (i) some measurements confirm a physical theory, (ii) its success confirms arithmetic, since otherwise (were it inconsistet) computations which rely on these measurements might prove erroneous, and (iii) the so confirmed reliability of arithmetic contributes to confirming the reliability of the theory of sets.

In this way, our belief in the existence of infinite sets, and other highly abstract entities, is only indirect; ultimately, it is based on perceiving some physically given, relatively small, sets. Contrary to Plato's fundamentalist belief that human cognition starts from clear and unshakeable vision of ideal objects. Such a vision can be approximated in the process of clarifying concepts, but it does not exist from the very start. How is this process developing – it is the subject of the next Section.

§3. Abstraction, idealization, generalization

§3.1. Gödel in his talks with Wang often emphasized close interrelations among the three cognitive phenomena listed above in this Section's title, and spoke also of intuition (insight). For instance, he says:

> Generalization and abstraction are closely related to idealization, which interacts with our intuition. [...] Indeed, we arrive at all our primitive concepts by idealization. What does idealization mean? It is the way you arrive at some concepts with different degrees of abstractedness. [...] You reach new primitive concepts by it. All primitive concepts are idealizations. — Wang [1996, Sec. 9.2.18 and 9.2.19, loc. 4728-30]

In the previous Sections of this paper I spoke mainly of abstraction, avoiding the use of the remaining two terms, since explaining their mutual relations would interrupt the run of discussion. Instead, I employed the term "abstraction" in a broader sense. Now it is in order to discuss the issue in greater detail. The most familiar example of idealization, found already in Plato, is that of ideal figures in geometry.

Abstraction. First let us consider a tridimensional physical object – a car wheel. When I look at its front from a short distance, I see only the surface. It is a dimensional object which I can think of (its size, colour etc.) as isolated, that is, without any thinking about the rest of the wheel. Then I *abstract* the surface from the rest. No surface can exist apart from the solid in question, but it does exist as a non-independent part of a whole; likewise there does not exist a solid without surfaces.

Idealization. The surface I see has the form of a circle. When I examine it as carefully as possible, e.g. with the magnifying glass, I find that the line of circumference is not ideally smooth; there are microscopic irregularities. This means that some points at the circumference are more distant from the circle's centre than other ones. Any time I examine the circumference of a wheel surface, I observe the same fact of small differences. Let us look at such a fact from the standpoint of an engineer who makes the design of wheels for the new type of car being produced. Obviously, his design ought to disregard possible physical difference of size, as those mentioned above, and be made according to the rule that every point at the circumference is equidistant from the point being the circle's centre. Thus his designing deals with an *ideal* circle.

Generalization. When our designer obtains a new task, to project wheels having a greater surface, as indicated by the producer, he uses the same mathematical formula to calculate the radius of the circle in question. This means that the concept of ideal circle, as defined in geometry, is *generalized* in the sense of embracing circles of arbitrary size; that is, of arbitrarily long radius. This implies that there is a potentially infinite number of cases falling under the concept of circle.

Insight. At every stage of the listed proceedings there occurs the activity of reason which is not identical with a sensory perception. Such a perception is but the point to start with, while the rest is due to activity that surpasses the range of sensory sphere. This is why it deserves the name of intellectual insight. For the same reason the category of *intellectual insights* extends over acts of idealization and of generalization.

§3.2. Now let us trace analogous stages in the case of arithmetical insights that lead to the concept of number. Introductory remarks on subject of arithmetical abstraction are found in §2.1. They comment the motto (of Section §2) in which Gödel, astonishingly enough, observes that there are sets which are physical entities, and are perceived by senses. Such sets provide the first grade of proceedings which lead to highest regions of abstraction.

Let us compare a pair of gloves with a pair of sockets. What differs them it is (ii) the difference in the properties of elements, and (ii) that in the pair of gloves each one has a function of its own, while in the case of sockets there does not occur any such otherness. What these pairs have in common, it is just the fact of their having exactly two elements. When we take into account all the pairs of any things in the universe, and *abstract* from the differences like those mentioned above, and focuss on their common feature of having two elements, then we do with the set of all pairs. This set is identified with the cardinal number two.

Such a process of *abstracting* brings forth *generalization*: "two" means the totality of all pairs. Having made such generalizing step, the imaginative mathematical mind makes a step towards idealization.

Note, first, that at the stage reached so far, the concept of two belongs to the universals: it extends over very many concrete individual pairs as its designata. Now let the set of pairs be conceived as single abstract entity, hence an individual. Analogously we replace the set of all tripples by the individual number three, and so on, including also operations which result in the numbers zero and one.

This is a process of *idealization* which for two reasons deserves to be called so. First, while a concrete pair, say, of gloves, falls under senses, while the individual number two does not fall. The latter presents to us as an *idea* of our reason.

Second, and more important, there is a striking difference between a pair of gloves and the number two. The concept of the former, like the other empirical concepts, is incurably vague, while the concept of two is perfectly *sharp* (a favourite term of Gödel, when considering mathematical notions). The vagueness of "pair" manifests itself in various ways. It is, e.g., difficult to say whether my pair of gloves continues to exist after I lost one of them in an unknown place. Or, when one of them will be halved into two discrete parts; and so on.

No such doubts appear with regard to the abstract object named "two". It is ideal as not likely to endure such defects of meaning. Compare this fact with differences between the notion of a physical wheel and the notion of an abstract geometric circle. As to the former, one may ask whether it remains a wheel after having been flattened in a car accident; or, after cutting off a small part of it. Such shortcomings do not touch ideal objects of geometry, arithmetic, etc. A reflexion on this difference in conceptual sharpness belongs to the core of Plato's thought; no wonder that it arises also in the modern mathematical Platonism, represented notably by Gödel.

As being ideal, and not given in a direct experience, such conceptions require another method of confirmation. Modern Platonists do not share Plato's belief in their privileged status of infallible visions. They are construed rather like entities which empirical hypotheses deal with; such conjectures ought to be confronted

with facts to be checked and, possibly, confirmed. This fallibilistic feature of modern rationalism, including its platonic branch, explains why the name "rationalism" is accompanied by the attributive "pragmatic"; fallibilism is a crucial point of pragmatism.

In order to be so tested, mathematical concepts should be embedded into a system of theorems, optimally, an axiomatic system. If a consequence of an axiom proves to be either the truth of another well-checked theory, or have significant practical applications, e.g. in technology, this heightens its degree of confirmation. The more numerous are successful checks, the greater becomes the degree of confirmation. The attribute of having many applications is duly called the *fertility* of the theory in question.

> History judges [mathematical] creations by their enduring beauty and by the extent to which they illuminate other mathematical ideas or the physical universe, in a word, by their fertility.

So says Gregory Chaitin [2006, loc. 184 in Kindle format] who continues Gödel's work both in metamathematical research and in Platonic philosophizing.

The term "fertility" does cover, indeed, applications to other abstract theories, and to empirical exploring the universe. This is connected with – as Chaitin says – "enduring beauty", since a theory being incoherent or denying factual evidence is an ugly phenomenon.

This idea of fertility repeats itself in Gödel, Quine, Frege, and other platonizing logicians. It appears also in main stream of Polish analytic philosophy, mainly with Ajdukiewicz, Łukasiewicz and Tarski (see §5.1). Its programmatic statement is found in the seminal Ajdukiewicz's lecture "Der logistische Anti-Irrationalismus in Polen" discussed in the next Section.

§4. The School's logical anti-irrationalism as a program for scientific philosophy

§4.1. *The School's manifesto by Ajdukiewicz, and the question of its distance to pragmatic rationalism*

The more the past becomes unveiled, the more we gain a temporal distance, and the more increases our knowledge of what happened in the meantime. Our picture of the School is nowadays much different from that existing in the period before the 2nd World War. Such a new picture makes us aware of an evolutionary process. Moreover, we enjoy a perspective on parallel processes, and this sheds a comparative light. Owing to such perspective, we discover some new features of pragmatic rationalism in the School's evolution.

The School's philosophical attitude in the thirties was presented in Kazimierz Ajdukiewicz's seminal paper "Der logistische Anti-Irrationalismus in Polen" [1935]. What he told then to the prominent international audience, was intended as a clear and convincing picture of the high achievements of Polish logic and philosophy. In fact, in that concise text Ajdukiewicz seems to have obtained such an effect, but this must have excluded an account of differences and controversies. [5]

Here is the crucial passage of Ajdukiewicz's manifesto, to wit the proposed definition of logical anti-irrationalism – quoted in the German original.

> Antirrationalismus [ist] das Postulat, (1) nur solche Sätze gelten zu lassen, die auf eine nachkontrolierbare Weise begründet sind, dadurch jede mystische Intuition oder Wesenschau ausgeschaltet wird. (2) Zweitens istes das Postulat der begrifflichen Klarheit und *sprachlichen Exaktheit* Der Wert der philosophischen Forschung nach keinen anderen methodologischen Kriterien bemessen wird, als nach jenen, welche für die spezialwissenschaftliche Forschung Geltung haben.
>
> Ausser diesen zwei Zügen ist noch speziell als (3) dritter die Aneignung der logistichen Begriffsapparatur und *der grosse Einfluss der symbolischen* Logik zu nennen — "Der logistische Anti-Irrationalismus in Polen" in "Erkenntnis" [1935, p.151].

It is in order now to try an exegesis of so influential text. The term "logistic" ("logistisch") has become obsolete in the meantime, and freely can be replaced by "logical" – provided we mean mathematical logic, that is, classical and non-classical logical calculi, and metamathematics as study of foundations of mathematics and logic. This is exactly what members of the School thought of, when speaking of logistics.

The term "anti-irrationalism" has been used instead of "rationalism" on account of the School's orientation toward scientific philosophy. When being scientific, philosophy opposes any kind of irrationalism – as tending to subordinate reason either to emotional forces or to some authorities (religious, political, etc.).

This does not mean that the School's members in the thirtees could realize the relation between their rationalism and the *pragmatic rationalism*, referred to in this essay's title. Then the term "pragmatic rationalism" did not exist yet. It appeared only in the second half of the past century, not without connexion with Quine's declaration of pragmatism. Quine admitted the ontological commitment of mathematics and natural science which so far was praised by rationalists alone, while disapproved by empiricists and nominalists.

Gödel in his postwar writings maintained somewhat similar pragmatic view, calling it *rationalism* or *Platonism*. Soon followed historical studies to embrace the whole of this phenomenon, and there appeared several varieties of pragmatic rationalism, connected with the names of Frege, Russell, Putnam, Kreisel, Chaitin, etc. Owing to such studies, done from

[5] To realize the content and scope of some important differences the Reader is advised to resort to Rafał Urbaniak's penetrative study "Leśniewski's Systems of Logic and Foundations of Mathematics", Springer. For instance, there are analysed in it varying approaches to the theory of definition, as proposed by Leśniewski, Łukasiewicz and Ajdukiewicz.

a temporal distance, we can interpret the School's views and trends in a new light. [6]

Item 1 in the quoted statement demands that (1a) only those statements be admitted in philosophy which are clearly stated intersubjectively testable, and (1b) not those which pretend to intuitively grasp the essence of a thing. Part 1b does not seem as obvious as 1a, when compared with the later Ajdukiewicz's [1958] claim that real definitions are indispensable for science, and they refer to universals. Essences are traditionally conceived as universals, hence that claim does not comply with Ajdukiewicz's [1935] renunciation of essences (more on this subject in §5.1). Items 2 and 3 require a more extensive discussion – see §4.3 and §5.1.

§4.2. *Some possible relations between philosophy and science. Kotarbiński's approach as different from the School's mainstream*

To fully grasp the meaning of the term "scientific philosophy", let us recall the title of Newtons typically scientific work "Philosophiae Naturalis Principia Mathematica" [1687]. This hints at the fact that in the 17-th century it would be nonsensical to consider collaboration between physics and philosophy, since physics was regarded as a philosophical discipline.

Only the successive steps of seceding by physics and other provinces from the empire of philosophy, have put on the agenda the issue of relations between philosophy and its seceded provinces. There are two main options to be considered by philosophers: either (A) total isolation, i.e., no mutual influences (existentialism, some kinds of thomism), or some kind of interactions. These can be the following.

Either
(B) the dependence of philosophy from sciences;
or
(C) the dependence of sciences from philosophy;
or else
(D) a symmetric mutual partnership.

The term "scientific philosophy" happens to be applied to option B and to D, though in different meanings, not compatible with each other.

As to B, it is characteristic of the positivist trend since Comte to the Vienna Circle, and especially Hans Reichenbach [1954] as the author of the book "The Rise of Scientific Philosophy". According to this view, philosophy should be guided by scientific results alone.

For instance, according to Reichenbach [1954, p. 48], the philosophical tenet that there are truths known a priori has been denied by the results of modern mathematics and physics. However, after a more careful inspection, one finds that this rejection does not result from mathematics and physics themselves, but from certain philosophy of mathematics and physics, to wit the philosophy of radical empiricism. Thus, paradoxically the contention which pretends to belong to option B, proves to be akin to C as well. [7]

A striking example of the dominance of philosophy over science (within the camp of radical empiricism) is found also in the School, to wit, with Tadeusz Kotarbiński. The razor of his reism, closely tied with empiricism, is so exceedingly sharp, that it is hard to imagine which part of physics could be saved. Let us take the following sample.

> It is not possible to accept such an interpretation of experimental data in which a particle of physical body would have, under certain conditions, to be identical with a wave. The noun "wave" is a special case of the general term "process" or "event", and as such, from the reistic point of view, yields nonsense when substituting for a name of a thing in an ultimate formulation. To say about a thing that it is a wave is equally nonsensical as say about a thing that it is a mode or a quality." — Kotarbiński [1979, p. 48].

The same, we guess, should be said of the wave theory of light of the classical physics. There was a long dispute between the wave approach and Newton's corpuscular theory in which experimental arguments were at stake (Young's experiment etc.). However, a reist does not bother about such arguments. He *knows a priori* (though he pretends to be an empiricist) that just Newton could have been right since particles of light are things (i.e. solids), while waves are no things. Should philosophers have such authority to control scientists?

In the question of the nature of light Newton could have been approved by reists. However, what about his formula of gravitation? It runs as follows.

> Every *point mass* in the universe attracts every other point mass with a *force* that is directly proportional to the *product of their masses* and inversely proportional to the *square of the distance* between them.

In this statement, corresponding to the symbolic formulation of Newton's law, the phrases italicized (by myself) are typically abstract terms. According to Kotarbiński, they have no sense unless translated into terms which refer to tridimensional bodies (solids) alone. Thus reists have to do their home work of providing such translations. It does not seem, however, that any such attempt has been done during the more than half century which have passed since uttering such claims by Kotarbiński.

§4.3. *On the mainstream of the School's scientific philosophy: the program and its accomplishments*

[6] See, for example, Rodych's [2005] under much speaking title "Are Platonism and Pragmatism Compatible?", also Chichara [1982], Wang [1996], Tieszen [2011].

[7] According to Ludwig von Mises, such a disregard of the role of *a priori* proves harmful also in the field of social sciences esp. economics. The astonishing success of von Mises' aprioristic economics in predicting (already in the twenties of the 20-th century) the failure of socialist economy, evidences poor performance of that "fashionable tendency in contemporary philosophy to deny the existence of any a priori knowledge" (we read in von Mises's "Human Action" [1963, p. 12]).

In comparison with the extremities of Kotarbiński's reism, the mainstream line of the School which was well-balanced, also in its claim that philosophy be scientific. Two documents express the program in question: Kazimierz Twardowski's [1929] speech at a jubilee of the Polish Philosophical Society, and Kazimierz Ajdukiewicz [1935] article as quoted above (§4.1). Both clearly represent the position labelled as D – symmetric mutual partnership.

Twardowski's statement runs as follows (translated ad hoc by WM).

> When elaborating scientifically certain views, once having belonged to metaphysics, particular sciences contribute to the scientific worldview. The same is aimed by authors of metaphysical ideas, insofar as they comply with scientific results. Then arises a mutuality: sciences take some ideas and assertions from metaphysical systems, and give them back to those systems at a higher level of scientific rigour. As far as this process does proceed, the philosophical worldview will more and more emerge from non-scientific and pre-scientific stages, and will be ever closer to the scientific views on the world and life. However, this process is never to end, just successive approximations are available in the course of evolution. — [1929, p. 383 in Twardowski 1965]

This statement is akin to Gödel's idea of the effective, but never-ending, progress of knowledge due to the feedbacks between philosophical insights and precise mathematical results. This fact contributes to the contention of the present essay that the School's logical anti-irrationalism fits into the Gödelian paradigm of rationalism (as discussed above in §2 and §3).

The mutuality stated by Twardowski reveals itself in the common history of philosophy and sciences. The Phytagorean metaphysical vision of the universe ruled by numbers inspired Galileo's program for natural science. Democritus' metaphysical atomism gave rise to scientific atomism. The idea of the universe's rationality lies at the bottom of empirical quest for mathematical laws of nature. The philosophical concept of causality guides the planning of experiments.

Gödel contemplated the Platonic landscape of objective mathematics, that is, the *infinite* domain oj mathematical entities, contrasted with the *finiteness* of mechanical computing procedures. This unveiled to him the insufficiency of mechanical procedures in number theory, and so led to the incompleteness theorems. This result, in turn, confirmed his initial Platonic insight – according to Twardowski's idea of mutuality, i.e. interaction between philosophy and sciences.

Ajdukiewicz's in his anti-irrationalist manifesto (see item 2, quoted above in §4.1) postulates for philosophy the same criteria of acceptability which hold for sciences. Too little is there said to be sure what criteria come into play; they are different for different sciences. In the search for a common denominator one can consider the feature of testability appropriate in the given discipline. Thus one may imagine that a philosophical theory should pass the test of fertility. It does pass when it results in a scientific theory which, in turn, proves successful according to criteria of the discipline in question.

§5. Varieties of rationalist attitude and scientific philosophising

§5.1. *Platonizing rationalism and its pragmatic orientation with Łukasiewicz, Tarski, Ajdukiewicz*

Gödel's rationalism was inspired by Plato, while pragmatism was guided by his own metamathematical research. Both features can be found in the School's mainstream. The thinkers listed in this Section's title had a lead in the School, due to their significant results and international academic recognition. Each of them was a rationalist and a pragmatist in his own way.

Jan Łukasiewicz — The one who most overtly confessed platonism was Jan Łukasiewicz. From an article on Łukasiewicz by Peter Simons we learn that he was earlier a nominalist but later, in the 1930s, he admitted his being now a platonist. [8]

To substantiate this statement, Simons quotes a passage from Łukasiewicz's paper "In defence of logistic".

> Whenever I work even on the least significant logistic problem, for instance, when I search for the shortest axiom of the propositional calculus, I always have the impression that I am facing a powerful, most coherent and most resistant structure. I sense that structure as if it were a concrete, tangible object, made of the hardest metal, a hundred times stronger than steel and concrete. I cannot change anything in it; I do not create anything of my own will, but by strenuous work I discover in it ever new details and arrive at unshakable and eternal truths. — "Selected Works", ed. L. Borkowski, p. 249

Simons makes a comment: *Rarely has the motivation for platonism been so eloquently stated.* Łukasiewicz believed that those abstract logical entities, which he encountered in his research, enjoy the highest degree of reality, even much greater "than steel and concrete". This is pure platonism. Then the statements which refer to these abstract objects must have the highest degree of certainty, no bit of fallibility could be attributed to them. Hence, as far as logical objects are considered, there is no feature of pragmatism in that domain of Łukasiewicz's thought.

However, we do not know his opinion about set-theoretical objects from the highest regions of abstraction. This might be a task for further study in Łukasiewicz's philosophy. Anyway, as for platonism, there is no doubt that with Łukasiewicz it had a strong representation in the School.

Alfred Tarski — There were in the School thinkers who adopted a pragmatic approach without wider philosophical considerations. In this category is found Alfred Tarski who in his practice clearly was a platonist in his set-theoretical investigations, but without any ver-

[8] Simons, Peter, "Jan Łukasiewicz", The Stanford Encyclopedia of Philosophy (Summer 2014 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/sum2014/entries/lukasiewicz/

bal declarations in favour of Platonism. This Tarski's attitude is stressed some recent studies concerning his views and practices. We find in them, e.g., the following comments.

> Over time Tarski came closer and closer to the outlook most fitting the scientific practice in general, namely a pragmatist outlook. By pragmatism I understand here simply an attitude primarily determined by the ways and needs of actual mathematical practice. Pragmatism rests upon the primacy given to use. — Sinaceur [2009, p.388, Sec. 3.5].

It was the pragmatic motivation which made of Tarski a platonist in his mathematical practice. Merely in practice, not in theory. However, this practice was extremely platonic, just for pragmatic reasons. The stronger existentially are assumptions, the more problems become tractable, that is solvable by proof or computations, and the shorter and simpler are proofs. To such highly efficient systems belongs Tarski-Grothendieck set theory, stronger than ZFC and other set-theoretical systems. This theory implies axiom of choice, the existence of inaccessible cardinals, etc.

Such pragmatic advantages are, presumably, the reason why this theory is willingly used for computer implementation, e.g. in the system Mizar. This device is systematically employed by the authors publishing in the journal "Formalized Mathematics – a computer assisted approach". [9]

Such rich systems are more endangered by the risk of antinomies than those being more cautious. Therefore they need an awareness of limits which would grant that the system in question does not surpass critical limits. Hence the advice given by Paul Bernays.

> It is desirable to find a method to make sure that the platonistic assumptions on which mathematics is based do not go beyond permissible limits.

This should be a task for further study: to investigate what measures were taken, or should have been taken by Tarski to safeguard his bold abstractions against antinomies.

Kazimierz Ajdukiewicz — There was in his philosophical career a time in which he seems to have distanced himself from platonism. In his program of logical anti-irrationalism (see §4.1) he mentioned the notion of *Wesenschau*, employed by Husserl in a Platonic vein, as an example of blameworthy irrationalism.

On the other hand, two decades later he defended the objective existence of universals, and this implies a defence of essences. Plato maintains that concrete beings acquire their essence through their participating in universals, that is, abstract forms separate from the objects of sense perception. Thus, insighting (Germ. Schauen) into an essence (Wesen) of a thing leads to the awareness of the corresponding universal.

Ajdukiewicz's defense of universals was entailed by his theory of real definitions. He relied in the mathematical practice of defining: any definition ought to be associated with the proof the defined object does exist and is unique. In real definitions – Ajdukiewicz claimed – it is a universal which is that existing unique object. Being aware of anti-Platonic attitudes in many academic circles, he forestalled attacks in the following, a bit ironic, way.

> There are people who see red when they encounter anything that savours of Platonic idealism. And the notion of real definition does savour Platonic idealism. If one calls the sentence "the square is a rectangle having four equal sides" a real definition of a square, then one refers to a univocal characterization of the genus square, i.e., of a certain universale. — Ajdukiewicz [1958, p. 125]

In the argument following the above statement, Ajdukiewicz demonstrates that the anti-Platonic contention of nominalists results in a contradiction (related to the paradoxes of intensionality). The strategy adopted by Ajdukiewicz in his theory of real definitions was a typical manifestation of his pragmatic approach.

Ajdukiewicz did not link Platonism and pragmatics in one phrase like "pragmatic Platonism". However, in his thinking these concepts were conceptually linked. He pragmatist creed has been explicitly stated in the opening adress of the conference "The Foundations of Statements and Decisions" held in Warsaw, 1961, with the presence of the international elite of philosophers and logicians (including such celebrities as Chisholm, Prior, Black, Kneale, Markov, Kalmar, Apostel, Supes, v. Wright, Lorenzen).

The title of the conference, intended to link cognitive (statements) and practical (decisions) features of scientific activity, was meant by Ajdukiewicz, [1965b] as the organizer and leader of the meeting, in the spirit of Peirce. This founding father of pragmatism was by him addressed with the following conclusion. *The above remarks emphasize the pragmatical point of view in the evaluation of scientific methods [...] which some prefer to refer to as methods of scientific decision-making.*

To account for Ajdukiewicz's understanding of the relation between pragmatism and rationalism, let me resort to a less official source, to wit my talks with Ajdukiewicz in 1961 and next years. I owed this lucky opportunity to the role of his assistant in the research project concerning the methodology of empirical sciences, run in the Logic Section of Polish Academy of Sciences. My contribution consisted in a critical analysis of the Vienna Circle theory of observational sentences, made from a pragmatic point of view (on that occasion, Professor told me about his recent conversation with Quine in which both observed a considerable kinship of their views).

The thesis of my contribution was to the effect that observational sentences have to presuppose ostensive definitions of some predicates. This, in turn, presupposes a strong rationalistic component: the intuitively grasped set-theoretical axiom of abstraction, what amounts to intellectual perception of a universal. This observation yields a special case of Ajdukiewicz's point that any real definition refers to a universal, namely

[9] See e.g. https://www.degruyter.com/view/j/forma

a case to hint at practical indispensability of ostensive definitions (as the basis of scientific empirical language), and thereby of real definitions, whose set includes those being ostensive.

Such cognitive processes can be also accounted in terms of Husserl's *Wesenschau*, which so loses a mysterious character, suspected by some philosophers. This concept was so seriously considered by the greatest representative of pragmatic rationalism, that a more extensive citation will be in order (note the much speaking title of the quoted work).

> Georg Kreisel has also noted Gödel's interest in Husserl in his article on Gödel in the *Biographical Memoirs of Fellows of the Royal Society* (Kreisel [1980], pp. 218-219]. Hao Wang has remarked in connection with Gödel's views in "What is Cantor's Continuum Problem?", that presumably Husserl's elaborate analysis of our perception of a physical object can be viewed as supporting Gödel's conclusion" (Wang 1996, p. 303) about the objective existence of mathematical objects and about mathematical intuition. He comments on another place that "perhaps Husserl's consideration of *Wesensschau* can be borrowed to support Gödel's belief in the objective existence of mathematical objects" (Wang 1996, p. 304). — Richard Tieszen, "After Gödel: Platonism and Rationalism in Mathematics and Logic". Oxford University Press, 2011. Location 112 in Kindle format.

To make the *Wesenschau* concept closer to our everyday experience and language, let us consider the following sentences.

1. y is longer then x
2. y is heavier than x
3. y is older then x.

The difference between x nd y can be expressed by a number of units of measure relevant to the category in question. We understand that in each case there is a number z of units – such that z added to x results in y. Now let us disregard, that is, abstract-from, the kind of entities, and kind of units, and leave only the numbers associated, respectively, with y, x and z. Let the same letters italicized denote mere numbers which express multiplicity of units. Then we obtain the following arithmetical formula in which "greater", that is ">", generalizes the predicates occurring in 1, 2, 3.

4. for any y, x: $y > x \Leftrightarrow \exists_z(z + x = y)$.

This formula may be said to possess a "metaphysical" significance, as it does express our insight into the essence of greatness. This insight obtained by abstraction reveals a universal that refers to the infinite set of pairs in which one element is greater than the other.

This leads to the moral that we all, as humans, are in fact pragmatic rationalists. We are rationalists since we have insights into essences of things (*Wesenschau*) at many steps of our everyday lives, as well as at many steps of doing science. And we are pragmatic who do not bother how much are they for this "vice" blamed by nominalists, reists, etc. Instead, we are happy with the fact that universals prove so useful in our thinking.

However, there is another moral yet, characteristic of the pragmatic approach. Insights into essences are not given at once, usually they require efforts in the course of cultural evolution. The generalization 4 could not be within the reach of primitive minds in a tribal environment though, presumably, its particular concrete instances, like 1, 2, 3 could have been perceived.

Nowadays we experience similar cognitive deficiencies at some sophisticated cases. Sometimes we need an insight and lack data to gain it, for instance at a such difficult case as the continuum hypothesis. The pragmatic approach amounts to the hope that a more penetrative analysis of relevant concepts should bring a solution liable to be checked by applications. Such a higher level of understanding will, in turn, give rise to new challenges to our intuitions, and so on.

§5.2. *Philosophical insights which first inspire science, and then are checked by science*

The cases discussed in this Section produce a list of unanswered questions – addressed to those who are to study the School's ideas and achievements from the historical distance of our time. The term "to check" in the title is used in the sense: "to verify by consulting a source or authority". As such authority to check philosophical insights acts empirical or mathematical science.

Here we deal with that interaction which meant Twardowski [1929] in his programme for philosophy. Sciences owe to philosophy vital inspirations, and repay with verification – with a result that is either positive or negative. At the bottom of that mutuality there is the pragmatic (hence anti-fundamentalist) claim that philosophical inspirations lack infallibility, and require checking as much as scientific conjectures.

Let us imagine that a philosophical insight has become successfully tested in science, and so obtained the status of confirmed generalization. Thus the ancient philosophical atomism has found its scientific articulation, for instance, in Newtons corpuscular theory of light, and then in the conception of matter having atoms as the smallest constituents.

Much worth of remembering are remarks on testing philosophical views, found in the excellent book by Einstein and Infeld "Evolution of Physics", Section "The philosophical background". The authors demonstrate how philosophical views, liable to be checked scientifically, derive from "ingenious figments of the imagination" – as they describe Democritus' vision of the universe. Such a vision, when combined with some scientific results, may evolve into a philosophical generalization, say, the modern atomistic theory of matter. This, in turn, contributes to a further progress of science, as described in the following paragraph.

> Philosophical generalizations must be founded on scientific results. Once formed and widely accepted, however, they very often influence the further development of scientific thought by indicating one of the many possible lines of procedure. Successful revolt against the accepted view results in unexpected and completely different developments, becoming a source of new philosophical aspects. — Einstein/Infeld [1938, p. 55].

This does not necessarily mean that an once accepted theory continues to be accepted for ever. On the con-

trary, it may be rejected in order to pave the way toward new revolutionary discoveries. Then, indirectly, there have to be abandoned philosophical insights or assumptions from which the rejected theory derives. However, such a successful revolt might have come to existence just owing to the fact that there existed something to be revolted against. And this is a real merit and contribution of a fallen philosophical insight. Such a fallibilist approach, concerning directly scientific theories, and indirectly philosophical insights which a theory comes from, leads to the fallibilist, and so, typically pragmatic, picture of philosophy.

In the materialist stream of ancient philosophy there was no idea of reality as a mathematical structure. This idea appeared with Pythagoras, and then Plato; they saw the universe as existing "under the number and measure" (so says an inspired by Plato verse in the Bible). This awareness reached the Middle Ages with its flourishing centres of Platonism (esp. Oxford), entartaining the thought that mathematics is the main key to apprehend reality. This paved the way to Galileo. Since his work, the science started to connect experiments with mathematically formulated teories in its endeavour of discovering laws of Nature.

The enormous efficiency of science urges pragmatists to think that such a great cognitive success evidences the truth of a successful theory. This is the very core of the pragmatic worldview. However, it encounters a strong opposition of the united camp of nominalism, materialism, instrumentalism, conventionalism; in the School, an ardent follower of this attitude was Kotarbiński.

The controversy is not decided yet. Thus there arises the task for pragmatic rationalists to analyse a relation between efficiency, that is, applicational success of a theory, and its being true. If such an epistemological analysis provides a proof that the former implies the latter, then the pragmatic rationalism with its Platonic component, will hold as a commonplace among philosophers.

§5.3. *Pieces of scientific philosophy done in the School. Their relation to analytic philosophy*

After the fairly extensive discussion, as that above, of the program of scientific philosophy cherished by the School, it is in order to ask: how this program was realized, and how much its accomplishment has proved successful? The answer will be sketchy and merely exemplary, as a full account would require a separate study.

The most telling is the case of Łukasiewicz for its enormous contribution to the problem of determinism, crucial for empirical science. First was the thorough study whose title would read in English "Analysis and Construction of the Concept of Cause" [1906]. In the beginning of 1920s he started his monumental work on many-valued logics (independently from Emil Post whose work appeared in approximately the same time). Łukasiewicz was interested in the philosophical, but crucial also in physics, controversy between determinism and indeterminism.

He believed that his multivalued system decides in favour of indeterminism. Thus initiated inquiry soon expanded itself into enormous area of research engaging the greatest logicians of the past century. In one of their applications (due to von Neumann and others) many-valued logics have become foundation for quantum mechanics. Łukasiewicz [1970] not engaged himself into so highly technical issues of physics, but as the initiator of the very idea of multi-valuedness, he should be praised also for indirect influences on so flourishing scientific fields.

This context gives us an opportunity to explain the role of *analytic philosophy* or (in another wording) *philosophical analysis* in activities of the School. Let us note that neither term appears in programmatic statements of Twardowski and Ajdukiewicz, though philosophy in the analytic tradition was being developed since Russell and Moore, from the beginning of the century. Perhaps the both leaders of Polish philosophy did not wish to resort to the concept which did not enjoy yet due clarity. However, in the course of time it became usual to speak of the School as a significant branch of analytic philosophy.

In fact, in one of its current meanings, the term "analysis" nicely applies to Łukasiewicz's many-valued logic. This fact should be considered jointly with the third item of Ajdukiewicz's speech on logical anti-irrationalism (see above §4.1): that the School acknowledges the great import of symbolic logic. Logic has proved an efficient tool of clarifying philosophical and scientific concepts.

This tool amounts to formalized axiomatic systems. When a theory gets axiomatized, its primitive key concepts, which so far lacked definitions, become precisely defined by the context of axioms. Peano's axiomatized number theory can serve as an excellent paradigm. The axioms of Łukasiewicz's logic define logical constants in such a way that the axioms become satisfied provided the existence of the third logical value; for the purpose of the present context it can be called *indeterminateness*. Thus an axiomatically clarified term can be used to define the philosophical notion of indeterminism.

A formal logical approach to the causation has been also elaborated by Stanisław Jaśkowski, the same who inspired by Łukasiewicz created in 1934 a system of natural deduction. Here we have a clear example of feedbacks among ontology, logic, natural science, and even Artificial intelligence, as convincingly explains Max Urchs [1994] – an author who penetratively studies and develops Jaśkowski's work.

It is not possible to comment here on all interactions of sciences with the School's scientific philosophy and analytical method. However there is one that cannot be missed even in such a concise survey. It is a meeting of logic and philosophy with linguistics and computer science. A theory which has arisen at that intersection has philosophical background in Frege's

ontology of object and function, and Husserl's idea of semantic categories.

The latter inspired Leśniewski. He took Husserl's idea to transform it into his own theory of semantic categories, to frame these categories in the hierarchy which could sucessfully replace the Russelian hierarchy of types.

Łukasiewicz's invention widely known as bracketless *Polish notation* is applied in computer science and cryptography where it renders valuable services in the form called *reverse Polish notation*.

The main idea of Polish notation, that of transforming a tree of parsing into linear order provides foundation for Ajdukiewicz's linguistic theory known as *categorial grammar*. It became the first algorithmically approached formal grammar long before Chomsky's work. It is best applicable to formal languages. However, it proves also useful in studying natural languages, since its limitations hint at those regions of ordinary langages in which formal methods must fail, and this stimulates a quest for alternative approaches. These achievements gained a wide international appreciation, as witnessed by the anthology "Categorial Grammar" [1988].

Let this snatchy survey give us a glimpse on how Twardowski's and Ajdukiewicz's idea of scientific philosophy was being realized. A closer approximation to such a survey would deserve a special research project, worth to be undertaken in the present generation of the School.

§6. Concluding reflections Who does belong to the Lvov-Warsaw School?

§6.1. *Aspects and degrees of being close to a philosophical paradigm*

The following are the author's concluding reflections; they should more clearly exhibit the content and scope of the term "Lvov-Warsaw School". This might be done with establishing criteria of being the School's *member*; such a goal is mentioned in the title of this Section. However, this is a distant goal to be attained, requiring a careful study.

Therefore, though I hint at it as a desirable happy end, I confine the present reflections to a preparatory, less challenging, job. It consists in some steps toward defining the relation of *closeness* to the paradigm of a philosophical school of thought or a philosophical tradition. Let us note that *belonging to a community* receives just two values: either you belong to the Polish Philosophical Society or do not belong. On the other hand, closeness is a gradable relation, having a number of degrees, and liable to be considered in various aspects.

So it is with the concepts of somebody's closeness to the community of Lvov-Warsaw School: one may be less or more close to its values, views, methods, achievements, and less or more close to this or to that of its leaders. After learning the degree of one's closeness to the community in question, we can try to establish how great should be the degree of one's closeness to decide about membership.

To exemplify how it may be hard to pass from recognizing closeness to recognising membership, let me take the case of mine as the one best to me known. I feel closeness to Ajdukiewicz's pragmatic rationalism from the last phase of his creativity, while I do not share his previous radical conventionalism, which on a par belongs to the picture of the School. I started from his results in writing on categorial grammar (e.g., Marciszewski [1988]). I had also some publications connected with Ajdukiewicz's program for the methodology of empirical sciences, and other ones resorting to his theory of real definitions. In these aspects there is an obvious closeness. However, I am not sure if this can suffice to feel a member of the School's second generation. The uncertainty is due also to the fact that I am close also to the epistemology based on the idea of computability, as developed by the Austrian School of economics (esp. von Mises and Hayek); I popularized its conceptions in some papers. But, on the other hand, I had a personal contact with Ajdukiewicz, and no such contact with Hayek; should such personal aspect prevail – in measuring closeness – over the Austrian School's intellectual influence?

It is in order to sketch the aspects in which one can consider somebody's intellectual closeness to the School's paradigm. However, the list given below does not include genealogical succession – a transitive relation defined as follows: if x is a descendant (i.e., pupil or assistant) of Twardowski, and y is a descendant of x, than y is a descendant of Twardowski. This criterion becomes intolerably fuzzy in the case of later generations, and – what worse – it may easily enter in conflict with any of the conditions listed below. In the academic case, a personal lineage does not ensure sharing ancestor's traits; an offspring's free choices are not controlled here by any set of genes.

Here are the aspects of closeness to the School, each labelled with a mnemotechnic abbreviation.

1. Val – intellectual and moral values: responsibility for the precision of thought and speech, spirit of companionship and mutual respect, the feeling of mission to promote logical and philosophical culture across society.

2. Col – efficient collaboration, and transmission of ideas from an earlier generation to the next.

3. Sci – scientific rigour in philosophy and its collaboration with sciences.

4. Met – methodological attitudes: pragmatism in the mainstream, and fundamentalism in the alternative mainstream.

5. Ass – the body of asserted judgements: rationalism in the mainstream, empiricism and nominalism in the alternative stream.

The first three items are shared by the whole community of the School in every generation. As for the remaining ones, they display a split between two par-

ties of the School, each opting for a position opposite to that of the other. One for pragmatic rationalism, the other one for fundamentalist empiricism; the former most explicitly represented by Ajdukiewicz, the latter promoted by Kotarbiński. This circumstance, often overlooked, proves important for estimating somebody's closeness to the School's paradigm. Two examples which follow should clarify this point.

§6.2. *An example of proximity to the mainstream's Platonic paradigm – Roman Suszko*

Let "proximity" mean a high degree of closeness. I am to consider two examples so selected that both in the highest degree represent the School's leading trend – attaining scientific preciseness in philosophy, and close collaboration of philosophy with science. Moreover, both examples havein common the mainstream's rationalist and Platonic attitude.

These resemblances are associated by complementarity; this grants yhis study a comparative valour. One of our exemplary characters – **Roman Suszko** (1919-1979) – acted at the intersection of philosophy and mathematics. The other – **Michał Heller** – is busy at the intersection of philosophy with physics and cosmology (with the due share of mathematics). At the same time, they represent two significant geographical affiliations, the former of Warsaw, the latter of Cracov – the main centers of scientific philosophy in Poland. Roman Suszko, chosen in this discussion for representing the Warsaw party, had also an academic tie with Cracow. There in 1945 he gained the diploma of master of philosophy. His dissertation, concerning the output of logic in Poland was supervised by Zygmunt Zawirski – one of the eminent representatives of the School, active mainly in its Cracov branch.

Suszki's doctoral dissertation, on analytic axioms and logical rules, defended in 1948 at Poznań University, was supervised by Kazimierz Ajdukiewicz. In the next years he enters the world of mathematics through having been habilitated by the Faculty of Mathematics and Natural Sciences at the same University; the title of dissertation "Canonic axiomatic systems".

His further career is divided into the field of mathematical logic in the academic units of Warsaw University and Polish Academy of Sciences, both run by Ajdukiewicz, and the field of mathematics in Institute of Mathematics of Polish Academy of Sciences; there he works on algebraic issues in collaboration with Jerzy Łoś. His collaboration with Ajdukiewicz comprised also establishing and running the journal "Studia Logica" which belongs now to the most important international journals on mathematical logic and related fields. Moreover, he was Ajdukiewicz's successor in running logic departments both in Academy and in University.

International Suszko's standing is witnessed by the list of the most respected journals publishing his results. There are among others: Fundamenta Mathematicae, Journal of Symbolic Logic, Colloquium Mathematicum, Synthese, Theoria, Logique et Analyse, Archiv für mathematische Logik und Grundlagen-Forschung, Studia Logica.

As for platonically coloured rationalism, it can be found with Suszko in at least two points. One is his favourite idea that the Cantorian set theory (in its various refinements) is the highest achievement of philosophical endeavours to create a formal ontology applicable to the whole universe. It is worth remembering that Cantor himself regarded his notion of set as the mathematical counterpart of Plato's philosophical notion of idea as an existing entity.

The other point can be read off from Suszko's [1964] inquiries into syntactic categories and denotations of expressions in formalized languages (in a volume offered to Kazimierz Ajdukiewicz). The study is based on Ajdukiewicz's categorial grammar with its infinite syntactic hierarchy of functors, analogous to Russel's types ladder. Suszko assigns each syntactic category the denotation; it is a function which belongs to the corresponding ontological category. This means a bold ontological commitment, since functions of ever higher order belong to the typically Platonic ontological landscape (blamed by Kotarbiński [1957, p. 158] as a philosophical illusion).

§6.3. *Another example of commitment into Platonism – Michał Heller*

Michał Heller, like Roman Suszko, eagerly follows Twardowski's and Ajdukiewicz's idea that philosophy needs a close cooperative interaction with science. As often stressed earlier in this text, it is the most characteristic postulate of the School, together with the tendency to heavily draw upon mathematical logic.

Here is no need to account Heller's academic career as it gets reported at another place in this volume. So at once we can start comparing his and Suszko's's proximity to the mainstrean of the School.

Heller's philosophical engagement in Platonism is more intensive than Suszko's. It involves a wide range of scientific issues in mathematics, physics, cosmology, logic – treated in numerous books and articles, many of them having been published abroad. It is not possible to give account of Heller's problems, results and ideas in a text like this, hence I limit myself to a shortened paraphrase of his statement about the endorsed by him Platonic position of significant contemporary scientists.

This position is to the effect that mathematical structures exist objectively and independently both of the physical world and of our cognition. There is a correspondence between the Platonic world and the physical world. It results into realizing that with inquiring into mathematical structures we learn about the structure of the physical world; in other words: the essence of nature is mathematical.

Heller [2011, p. 15] clearly sympathises with such strong adherents of Platonism as Kurt Gödel and Roger Penrose. The following quotations in his book seem to express the views of his own.

Gödel [1989, p. 137] wrote: "It seems to me that the assumption of such [sc., mathematical] objects is quite as legitimate as the assumption of physical bodies."

Penrose [1989, p. 116] wrote: "My sympathies lie strongly with the Platonistic view that mathematical truth is absolute, external, and eternal, and not based on man-made criteria; and that mathematical objects have a timeless existence of their own, not dependent on human society nor on particular physical objects."

Thus Heller's proximity to the Platonic position of the School's mainstream is as firm as it may be wished by most engaged Platonists. As for the second component of the mainstream, the methodological pragmatic attitude, it is not so explicitly articulated. Being just an amateur in studying Heller's output, I may be unaware of some sources relevant to this matter, hence the issue has to remain open in the present discussion.

However, it can be guessed that so close ties between mathematics and physics, as seen by Heller, entail the following: the fallibility of an empirical science, as physics, could entail fallibility of mathematical theories employed as models of a physical reality. Such, for instance, seems to be Gödel's position which, presumably, might be shared by Heller.

I like adding a message which I owe to personal communication by Paweł Polak reporting on some Heller's statement at a conference: that at various levels of reality one should apply systems of logic relevant to the level in question. A typical example: the use of a multi-valued logic at the level of quanta for which the two-valued logic does not prove adequate. If so, a system of logic which operates on so and level would be either confirmed or weakened in dependence from the fate of the empirical theory in question.

The issue of applying logic at the quantum level of reality was thoroughly considered by Zygmunt Zawirski (1882-1938) – one of the most eminent representatives of the School. His academic career was connected with Lvov, and mainly with Cracov. He defended his habilitation thesis (1924) on the axiomatic method in natural sciences at the Jagiellonian university, where he lectured since 1937 until his death.

Zawirski's studies in application of multi-valued logics and the probability theory much contributed to the cooperation between philosophy and natural sciences (e.g, in Zawirski [1935]). They perfectly fitted into Twardowski's and Ajdukiewicz's program, and at the same time perfectly complied with the Cracov tradition of doing philosophy in a close contact with natural sciences. What in Lvov and Warsaw was postulated, in Cracov was being efficiently accomplished by a host of eminent scholars, such as Smoluchowski, Natanson, Metallman, Zawirski, Gawecki – the heroes of the three volumes of the book by Heller and Mączka [2007].

Thus the Cracov stream of philosophy-of-science and philosophy-in-science has merged with the mainstream of the School initiated in Lvov and continued in Warsaw. In Warsaw under the impulse of Ajdukiewicz flourished the methodology of empirical sciences practised in a formal manner by Marian Przełęcki, Klemens Szaniawski, Ryszard Wójcicki, and other expert researchers. To Heller goes the merit of a complementary approach: his knowledge of philosophical and logical aspects of methodology gets supported by personal experiences in the theoretical physicist's workshop.

Hence the scientific philosophy in Poland enjoys two fertile traditions cultivated in two centers of Polish scientific culture. So far they remain in a mutual friendly appreciation, and every now and then experience fruitful meetings. However, the time seems to be ripe for a much closer collaboration. The so united forces should form the potential to dramatically speed up the aspiring project of scientific philosophy.

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