CHAPTER SIX

The Predicate Calculus

1. Subject, predicate, quantifiers

1.1. *In the beginning was the Word* teaches us the Gospel according to St. John. But which kind of word? The Name, destined to play the role of the subject in a sentence, or rather the Predicate? And, if we opt for the Name, should it be the Proper Name, as in 'John the Evangelist', or rather the General Name, as in 'Man'? These grammatical distinctions are of consequence for logic, too. In discussing them I shall take advantage of a grammatical theory suited for the language of modern logic.

The dilemma as worded above in the biblical quotation can be expressed in the theory called **categorial grammar**. Though the adjective 'categorial' refers to a feature common to all grammars, as each of them offers a categorization, in this case there is a special reason to take advantage of this term. Namely, the theory in question is built on a distinction which makes it possible to develop a calculus of categories. It is the distinction between **basic categories** and **derived categories** of expressions. This terminology implies that some categories are derivable from other ones; at bottom there are those which are not derivable themselves but provide the rest with the basis for derivation. The set of rules to define valid derivations, namely those which result in syntactically coherent (i.e., grammatical) expressions, forms the calculus characteristic of such a grammar.

Thus, when metaphorically asking what kind of word was at the beginning, we raise the question about categories to be acknowledged as basic. The answer given in terms of categorial grammar allows us to clearly observe the grammatical difference between the language of Aristotelian logic and that of modern logic: in the former there is the basic **category of general names** which does not occur at all in the latter, while in the latter there is the basic **category of individual names** which does not occur at all in the former. As for predicates, in the standard version of predicate logic called first-order logic, there is an infinite set of predicate categories, each of them being directly derivable from the category of individual names.[1]

The set of predicate categories is infinite since a predicate can be derived, that is to say, formed out of one (individual) name, or two, three, four names, and so on (theoretically) to infinity (in practice, though, we deal with a finite and rather a limited number). The names (which in this context are always construed as *individual* names) giving rise to a predicate are called its **arguments**. Thus there are one-argument (one-place, unary), two-argument (two-place, binary), three-argument (three place, ternary), etc., predicates. Let these categories be exemplified by the following predicates:

'... is a cat', '...is a descendant of ...', '... lies between ... and ...',

respectively, where each string of dots is to be filled up by a name, and the number of such strings corresponds to the arity (i.e., the number of arguments) of a predicate.

An expression belonging to a derived category is called a **functor** by analogy to a mathematical function sign which is also accompanied by a sequence of arguments. A functor is seen as an 'active' element in forming a compound expression out of simpler ones, hence the classification of functors is made both according to the category of expression which a given functor makes up and the categories of expressions from which the compound is made.

The syntactic description of predicate logic involves two basic categories, viz., that of names and that of sentences; let them be

---

[1] In logics of higher orders (see 3.3 below) there are predicates whose derivation from the basic category of individual names is not direct; directly a predicate of order n derives from predicates of order $n-1$, i.e. those of which n-order predicates can be predicated. A brief information about higher-order logics is found in 'Predicate logic' by W. Marciszewski in *Logic* [1981]; this author gives an introduction to categorial grammar in the same volume, while its more advanced discussion is found in Buszkowski, Marciszewski, van Benthem (eds.) [1988].

symbolized by the indexes 'n' and 's', respectively. A unary predicate forms a sentence out of one name; this fact is conveniently symbolized by the compound index 's:n', where the letter before the colon (sometimes a fraction line is used instead) hints at the category of the compound expression, and the letters following the colon (or, written under the line) hint at the arguments from which the compound is produced. Correspondingly, a binary predicate is indexed as s:nn, while the expression resulting from a sentence (such as an argument) transformed into a name should be indexed as n:s.[2]

The syntactic fact that predicates in modern logic do not belong to basic categories does not undermine their import in the semantic dimension, that is the role of conveying information. This role, which in traditional logic is played by a general name, functioning either as the grammatical subject or as the predicate of a sentence, is in modern logic taken over by predicates; the (individual) names act only as elements necessary for syntactic construction, while the informative function belongs wholly to predicates.

1.2. Let us more closely examine the fact that the predicate alone, and not in collaboration with the grammatical subject, is to furnish information about the state of affairs referred to by the sentence in question. This semantic difference entails a radical difference of the syntactic structures of sentences.

To explain this issue, let us consider the following sentence (in which the subject phrase is underlined while the predicate is in the slant type):

α: <u>Every man who names me traitor</u> is *lying like a villain.*

The above sentence is a paraphrase (unfortunately, a clumsy one) of the following exclamation found in Shakespeare:

---

[2] The index n:s indicates, e.g., the category of the functor 'that', as brought forward by the following analysis. 'It rains' is a sentence, and 'that' transforms it into the name 'that it rains'. That the latter is a name follows from the fact that it can be used as an argument of a sentence-forming functor, as 'is bad', to result in the sentence 'That it rains is bad'. It is not the only syntactic interpretation of 'that'; this particle has more philosophically and logically interesting intepretations (this problem is extensively discussed by Marciszewski [1988]).

β: Whatever in the world he is that names me traitor, villain-like he lies.

While α is a single sentence, due to the fact that it involves only one occurrence of the verb 'is', β is composed of two sentences, as seen in the double occurrence of 'is' accompanied by double occurrence of the subject 'he'. Such a subject, as being only a pronoun without any specific content, does not convey any information, hence the task of carrying the whole information is performed by the predicates 'names me traitor' and 'villain-like lies'.

Now let us replace the personal pronoun 'he' by the variable x ranging over the entire universe (as expressed by the phrase 'whatever in the world'), and let us make the conditional structure of β more explicit through introducing the connective 'if...then'. Thus we obtain the following (again, underlining the subject and slanting the predicate):

γ: For any x if $\underline{x}$ *names me traitor*, then $\underline{x}$ *villain-like lies.*

The form γ is the typical transformation of a universal one-subject sentence, in traditional logic reckoned among the so-called **general** or **categorical** statements, into a conditional sentence in which the whole information is contained in predicates.[3] English, like other natural languages, can use both forms, conditional as well as categorical, while in the logical languages examined here, only one of these methods is adopted, namely the categorical α-form in traditional logic (cf. Chapter Four, Subsec. 1.2, etc.) and the conditional γ-form in modern logic.

Both forms should be carefully examined from the rhetorical viewpoint, for two reasons at least. First, because we practise rhetorics in natural languages, which employ both structures; second, there are serious philosophical motivations behind either of these logical structures. Those philosophical issues, in turn, are

[3] The terms 'sentence' and 'statement' are used interchangeably, and so are the terms 'general statement [sentence]' and 'categorical statement [sentence]'. As to the latter pair, its first member was introduced in Chapter Four, Subsec. 1.2, where the adjective 'general' was more convenient as hinting at the generality of the subject discussed in that context. In the present context it is the adjective 'categorical' which proves more convenient as opposing the adjective 'conditional'.

concerned with cognitive processes of consequence for the art of argument.

There is an ambiguity in the meaning of the term 'predicate' which should be removed before we proceed with our inquiry. The ambiguity appears in the sentences made out of two terms and copula, such as "She is an artist". The parsing of such a sentence can result either in

she [*is an artist*]

or in

she is [*an artist*]

(as above, the underlining marks the subject, while the slant, here combined with brackets, marks the predicate).

The first parsing agrees with a standard grammatical rule, the second follows the usage of traditional logic in which the copula appears between the terms, the term preceded by it being called 'predicate' (Latin *praedicatum*). To avoid ambiguity, I reserve the expression **predicate** for the case of twofold partition, to comprise copula and the name that follows it, and adopt the expression **predicate term** to the case of the threefold partition in which the verb 'is' is predicated of two entities, one of them denoted by the subject term, the other one by the predicate term. This way of speaking is usual in describing the sentence forms of traditional logic.[4]

1.3. As said with reference to the examples α and β, when the subject term is a pronoun or a variable, it is not able to convey any information about the entity which the sentence in question refers to. In such cases, the whole task of carrying information is performed by the predicate term.

To simplify the matter, let us consider such a simple sentence as 'he is a liar', called atomic by logicians. An **atomic sentence** is one that consists solely of a predicate (as 'is a liar') accompanied by the appropriate number of **terms**, i.e., individual expressions, either constants or variables; in a natural language the role of variables may be played by pronouns (e.g., 'he'). Accompanying expressions are called **arguments of the predicate** in question. The appropriate number of arguments depends on the meaning of the

[4] See, e.g., Kneale and Kneale [1962], p. 65.

predicate accompanied by them. The meanings of '... is a liar', '....is big', '... walks', etc., hint at one entity to be predicated of, hence predicates of this kind are called **one-place**, or one-argument, or unary predicates.

The class of **two-place** (binary) predicates can be exemplified by expressions like

'... is the father of ...',
'... is a friend of ...',
'... is bigger than ...',
'... dances with ...',
'... precedes ...';

while the expression

'... lies between ... and ...'

is an example of a **three-place** (ternary) predicate. Theoretically, the number of places (as marked by blanks in our examples) is unlimited; practically, it conforms to the needs of communication and capacities of our minds.

It should be noted that the triples of dots used above as blanks, to indicate the number of argument places, perform the same role which is characteristic of variables; the latter also mark free places to be filled in. If one prefers the blanks technique, then in the case when the same object is to be referred to more than once, the place for the expression referring to it should be marked with identical blanks, say a dash '–' while blanks for other arguments should have a different shape. Then one would put '... precedes –' to express the same as the expression 'x precedes y' does, while '... precedes ...' would correspond to 'x precedes x'. Because of the practical inconvenience of blanks technique, we use rather letters of various shapes, called variables. However, the analogy with blanks should be remembered to properly understand the use of letters as variables; in logic we use letter arguments also for other purposes, hence seeing variables as blanks helps to avoid misunderstandings.

1.4. The discussion concerning variables as blanks was to shed light on the nature of predicates, and their partition according to number of arguments. Now it is in order to discuss the functioning of arguments. There are two methods to fill a blank with an expression referring to an object; if all blanks are filled, the predicate becomes a sentence.

One of the methods consists in using a proper name. Let the place marked with 'x' in 'x is a liar' (or, equivalently, 'he is a liar') be filled with the proper name 'Epimenides' to result in the sentence 'Epimenides is a liar'. In this context we can see the double role of pronouns. They may be used either in the function of variables or blanks, as shown in the examples above, or as substitutes for proper names. If in the presence of Epimenides one hints at him and says 'he is a liar', then 'he' means 'Epimenides'. It can be said that such a procedure transforms the pronoun (or, variable) 'he' into a proper name which is made out of this pronoun and that situation which involves the gesture of hinting and the object hinted at.

There is an English verb which fittingly describes what is going on in a case like that of referring to Epimenides through 'he'. It is the word 'to bind'. In the hinting procedure the expression 'he' becomes *bound* to a definite person, say, that of Epimenides. Due to the binding, 'he' changes its linguistic function, it is no longer a variable, in spite of preserving the same physical shape. The same holds when a letter, say 'x', is used as a variable. This can be better seen in a generalized form of binding which is the following.

Let us consider the view of a pessimist to the effect that 'everybody is a liar', or (equivalently) 'all people are liars'. In a half-symbolic language it can be stated as follows:

δ: *for any x holds: x is a liar.*

Now 'x' has a different meaning than inside the predicate 'x is a liar'. In δ, 'x' refers to any entity, i.e., to whatever in the world (in accordance with Shakespeare's phrase in β, in 1.2), while inside the predicate in question it has no reference at all, lacking any meaning in the same way as an empty space. Hence the phrase 'for any x holds' (or, shorter, 'for any x') performs the role of binding the variable 'x', i.e., of transforming it into a symbol which refers to something. To distinguish these two roles, that of a blank and that of a symbol having reference, logicians decided to employ the terms a **free variable** or **real variable**, and a **bound variable** or **apparent variable**, respectively.

The adjectives 'bound' and 'free' are currently used nowadays, while the other pair is going out of use. However, the old-fashioned terms are desirably suggestive. The phrase 'real variable' is to remind us that only non-bound symbols, i.e., those functioning as blanks, are genuine variables, while binding deprives them of that function, so that they occur as variables only apparently, on account of having been left in the same shape as before binding.[5]

There is another way of binding a variable, viz. with the phrase *for some ...*, or (equivalently) *there is ... such that* (i.e.., satisfying what follows; instead of 'there is' one may say 'there exists' or 'exists'). With this phrase we obtain sentences like that:

ζ: *There is x such that: x is a liar.*

If our universe ('whatever in the world') is defined as the set of all people, that is, it does not involve apes, angels, etc., then ζ simply means: 'some people are liars'. The use of the plural in this translation is only for stylistic reasons, as the same can be said with the sentence: 'at least one man is a liar' (this phrasing should be added to those listed above, as being another stylistic variant).

The phrases used in δ and in ζ (including their synonyms), apart from their common task of binding variables, have another feature in common. In a vague but unquestionable way they deal with some quantities, namely numbers of objects. *For any* means 'as many entities as are in the universe in question', while *for some* means 'not less than one'. Even if not very precise, they hint at certain quantities, and this is why these phrases and their symbolic counterparts in formulas have been called **quantifiers**. It has been shown in the above discussion how quantifiers interplay with predicates in forming sentences. First, variables are added to predicates as their arguments to result in the structure of a

[5] David Hilbert used different letter forms to distinguish these kinds of symbols: the lower-case letters from the beginning of alphabet for free variables, and those from the end for bound variables. This recommendable precision has not been followed by other authors, who regarded that the context is sufficient to prevent ambiguity. Common practice prevails, and therefore I do not follow Hilbert's way here, but it is worth remembering that through this simplification a useful notational device is lost.

sentential formula (called also 'propositional formula', or 'open sentence'), next come quantifiers to form a full sentence, which, if we need to clearly distinguish it from a formula, is called a **closed sentence**.

Quantifiers belong to the category called **variable-binding operators**. There may be other quantifiers besides the two discussed above, for instance 'there is exactly one', 'there are not less than two', 'there are infinitely many' (for various kinds of infinity), and so on.[6] However, in its standard version logic is content with two variable-binding operators which form sentences, namely the **universal quantifier**, to which a variable owes its referring to the whole universe, and the **existential quantifier** — that to which a variable owes its referring to at least one entity in the universe.[7]

The occurrence of these two categories, predicates and quantifiers, accounts for the fact that there are two equivalent names for modern logic, viz., **predicate logic** and **quantification logic**. In this essay, the term 'predicate logic', or 'the predicate calculus', is preferred as one that directs our attention to some comparisons between modern and traditional logic, the latter being seen as a logic of names.

2. Quantification rules, interpretation, formal systems

2.1. The question of how to use a word or a symbol amounts to the question of its meaning, while meaning is what is being brought in by a definition. This general comment is in order at the start of the present section, for it leads to a moral that is both logical and rhetorical. The moral is addressed in particular to those who in the name of scientific rigour are forever demanding 'precise' definitions, meaning by this statements in the lexicographical form 'A means so-and-so'. Such people used to react with a contemptuous 'I do not understand' when a speaker was unable to recite such an 'exact'

[6] More on this subject in *Logic* [1981], "Quantifiers" by S. Krajewski.

[7] Besides quantifiers, there are variable-binding operators which play another syntactic role: when binding a variable, they transform a predicate not into a sentence (as the quantifiers do) but into a name. The latter are important from the rhetorical angle as helping to make logic closer to every-day arguments, hence special attention is paid to them later (Subsec. 3.4).

definition. However, many important notions, e.g., those occurring in the axioms of a theory, cannot be so defined (unless one commits an infinite regress). In such cases, an efficient method of defining consists in showing words in use, and this practice should also be followed in everyday arguments and discusions. A convenient term for this kind of definition is **implicit definitions**. Logic and mathematics supply us with paradigms of that procedure, and the case of quantifiers is in this sense paradigmatic.

The use of a quantifier consists in either adding it to a formula or omitting it in the course of inference. If this is done in a **truth-preserving** way, that is to say, a true formula remains true after having been so transformed, then the inference in question is logically valid. The listing of truth-preserving uses of a quantifier amounts to a definition explaining its meaning; such a list can be regarded as a concise introduction to predicate logic. When reading a classical text consisting of proofs, say Euclid, one clearly sees that such operations were ubiquituous in the practice of reasoning, even if not codified in any logical theory available at the time in question. Such codification was first accomplished in contemporary logic.

Before stating the rules of using quantifiers, let me introduce a convenient notation.[8] The variety of phrases used for wording the **universal quantifier** in a natural language will be represented by the symbol '$\forall$', followed by the letter to indicate the variable (within the succeeding formula) being bound by this quantifier. Let $\Phi(x)$ be any formula containing 'x'.[9] The universal quantifier forms a formula like this:

$$\forall_x \Phi(x)$$

[8] There are other notations for quantifiers (see *Logic* [1981]); the one chosen for present purposes has the merit of being suggestive inasmuch as it depicts the universal and the existential quantifiers as stylized abbreviations for 'All' and 'Exists', respectively.

[9] As usual in mathematical practice, the letters from the middle of the Greek alphabet will be used to denote any formula whatever, without hinting at its content and structure; the only relevant information is to the effect that the formula includes the variable being bound by the quantifier prefixing that formula.

Analogously, various ways of expressing that an object is such-and-such, will be unified with introducing the symbol of **existential quantifier** which forms the following formula:

$$\exists_x \Phi(x)$$

For each quantifier there are rules on introducing it into a formula and eliminating it from a formula. In the rules there occur symbols acting as proper names of objects satisfying the formula in question. These will be, so to say, dummy names, inasmuch as we do not deal with a concrete domain from which definite things might be picked up, but rather with a schematic representation of any possible domain. The symbols which are to function as schematic, or indefinite, proper names are lower-case letters beginning the alphabet, viz., a, b, c, etc.

2.2. We eliminate the universal quantifier when we transform a universal proposition into a singular one, that is into a proposition referring to one from among the instances of the universal formula. This means that together with dropping the quantifier we replace the bound variable by the proper name of an entity satisfying the formula in question. The corresponding transformation rule runs as follows:

[**EU**] from $\forall_x \Phi(x)$ infer $\Phi(a)$,

where a, as explained above, is the name of an arbitrary object satisfying the formula Φ. Let the inference be illustrated by taking '$x = x$' for '$\Phi(x)$'; then from '$\forall_x (x = x)$' there follows '$a = a$'. If one takes into account a definite domain, say that of natural numbers, then the above universal identity results in '1=1', '2=2', etc. The abbreviation which is to function as the label of this rule stands for **Elimination of the Universal quantifier**.

The next rule to be considered is that of **Introduction of the Universal quantifier**. If a formula is satisfied by every entity in the domain in question as is, e.g., '$x = x$' in the domain of all things, then it is allowed to be transformed into a universal proposition, that is to be prefixed by the universal quantifier. The assertion of its being so universally satisfied is expressed by taking it as a premise of inference. With this comment in view, the rule is to be stated as follows:

[**IU**] from $\Phi(x)$ infer $\forall_x \Phi(x)$.

Analogical operations hold for the other quantifier. **Elimination of the Existential quantifier** means, as in **EU**, dropping the quantifier and replacing the respective variable by a proper name, but with the proviso [p] *that the same name was not earlier introduced with eliminating the existential quantifier in another formula.* Should one ignore this restriction, then a false proposition might result from applying this rule, e.g., to the formulas '$\exists_x$(*x is a liar*)' and '$\exists_x$(*x is a saint*)' after replacing 'x' by the same name in both formulas.

[**EE**] from $\exists x \Phi(x)$ infer $\Phi(a)$, provided [p].

The last rule to be listed sheds light upon what one calls the existential import of names as discussed earlier (Chapter Four, Sec. 2) and in this Chapter (Subsec. 3.4). It is the rule of **Introduction of the Existential quantifier** that runs as follows:

[**IE**] from $\Phi(a)$ infer $\exists_x \Phi(x)$.

2.3. The statement of rules given above is most general, covering all possible structures of the formulas prefixed with quantifiers. The Greek letter $\Phi(x)$ represents any formula whatever, if only that formula contains the free variable 'x'. It may be the simplest sentential expression involving a one-place predicate, as 'Px', or one with a more-place predicate, as 'Rxy', or else an expression with more predicates combined by connectives (i.e., 'and', 'or', 'if...then', etc., e.g., '$\forall_x$(Px *and* Ryz)'), as well expressions containing more quantifiers (either before the formula or inside it), as, e.g., '$\forall_x \exists_y$(Pxy *and* $\forall_z Qz$)'; another way of making a structure more involved depends on denying (i.e., prefixing with a symbol meaning 'it is not the case that') either the whole formula or some of its components. This way of representing arbitrary formulas ensures the desirable generality of the inference rules presented above.

As a rule, the structure of a formula is determined by the interplay of word order and punctuation signs; only in the so-called Polish notation, devised by Jan Łukasiewicz, does the syntactic structure depend on word order alone, but this theoretical merit is achieved at the cost of perspicuity. Hence, in practice, we benefit from both means of structuring expressions, using parentheses as the punctuation signs (no other punctuation devices are needed, since parentheses prove sufficient to define the scope of the symbols involved).

In particular, parentheses hint at the scope of a quantifier, and so tell us which variables are bound by the quantifier in question. For instance, the two following conditionals have different meanings because of structural differences resulting from the scope of the quantifier:

[1] $\forall_x Px \to Qx$,
[2] $\forall_x (Px \to Qx)$.

In [1] only the antecedent occurs in the scope of the quantifier, while in [2], as marked by the parentheses, the scope extends to the end of the formula.

2.4. At last, the very notion of a formula should be defined. This Subsection is to deal with the concept of a formula, with interpreting formulas by reference to the universe of discourse, and with distinguishing between formal and interpreted languages.

The concept of a sentential formula of a definite language, in short, a **formula**, is an expansion of the concept of a **sentence**; in logic, the latter term usually stands for an expression which has the grammatical form of a sentence, and involves no free variables. Every sentence is a sentential formula, but an expression with free variables is no sentence; it can be transformed into a sentence either through binding all variables or through replacing all free variables by proper names.

More systematically, the concept of formula is defined as follows. We start from defining the set of **atomic formulas**. For this purpose, we must have the list of predicates (let them be symbolized by some upper-case letters) as well as the list of individual variables and names (i.e., proper names) in the language in question. The elements of the latter category, comprising symbols for individuals, are briefly called **terms**. Atomic formulas are obtained through juxtaposing predicates and terms in the following order: first a predicate, then as many terms as result from that kind of predicate: one term follows a one-place predicate (e.g., Pa, P_1x, Qy), a pair of terms — a two-place predicate, i.e., referring to a two-place relation (Rab, Rax, Sxy), a triple — a three-place predicate, i.e., referring to a three-place relation (e.g., R_1xyz, as in 'x lies between y and z), and so on; often the terms, as arguments of the predicate in question, are put in parentheses and separated by commas, e.g., $R_1(x, y, z)$, but no ambiguity arises if one abandons such punctuation marks.

Once having obtained the set of atomic formulas, we define the set of compound ones, the composition being of two kinds. One of them consists in prefixing a formula (either atomic or earlier obtained from atomic ones) with a quantifier, while the other involves combining earlier existing formulas by means of sentential connectives, such as 'and', 'or', 'if...then', the latter set including also symbols which may precede formulas, such as a negation sign (to be read 'it is not the case that'). The number of such connectives varies depending on certain linguistic conventions; we are not bound to make decisions now in this matter, it is enough to note that a precise definition of a formula of a given language takes into account all such symbols accepted for the system in question.

Having thus settled the syntactic question of producing atomic formulas from predicates and terms, and producing more compound formulas from less compound ones, we can state a crucial semantic problem: what do such formulas refer to? For example, what is the formula '$\forall_x(x = x)$' about? The answer sheds light on the turn brought about by modern logic in our thinking about language. Both in natural languages and in traditional logic, a proposition possesses a meaning without any reference to the whole domain of thought being presupposed in the discourse in question. In modern logic, though, a formula does not receive any interpretation until one defines the set of objects to be referred to by individual variables. Such a set is called the universe of discourse or, shorter, the **universe**; it is said to provide the language with **interpretation**. Without interpretation a language is only a **formal system**, that is a set of rules concerning formation and transformations of certain strings of symbols which do not denote anything.

Such a separation between syntactic and semantic components of a language has far-reaching consequences. It makes it possible for a computer to effectively manipulate linguistic symbols without any need of simulating their understanding; after these manipulations are produced it is up to a human being to give them meaning. And, once having understood the nature of a computer, we are better able to make illuminating comparisons between machines and organisms, especially human bodies as producing texts and dialogues.

The means for producing formal systems when combined with the concept of interpretation enable us to introduce the concept of a *formalized* system, or a formalized theory. Imagine a system in which theorems are proved in such way that each inference step is justified by formal inference rules, i.e., rules which take into account only the physical shape of formulas. Imagine that, at the same time, the system is given an interpretation, hence its terms refer to objects in a definite domain. For instance, the variables of Boolean algebra constructed as a formal system become interpreted by assigning to them objects which belong to the universe of classes, and the operation symbols are interpreted as operations on classes; then we have to do with a formalized theory, i.e., one sharing the inferential rigour with formal theories and at the same time being intepreted. Owing to this combination, a formalized theory is manageable both by humans and by computers, and at the same time it has a meaning and importance for humans. This is exactly what Leibniz dreamt of, with the limitation that there are theories, as important as, say, arithmetic, in which it is not possible to prove all their truths in such a purely mechanical way; thus their formalization can be only partial.[10] Nevertheless, formalization remains a tool which essentially contributes to the use of computers as devices assisting human intelligence.

3. Predicate logic compared with natural logic

3.1. The presentation of symbolic logic in this and in the preceding chapters provides us with sufficient material to introduce the

[10] This famous limitative result is due to Kurt Gödel (1906–1978) (Gödel [1931]). More information on this and other limitative results of modern logic can be found in *Logic* [1981], articles by S. Krajewski: 'Completeness', 'Consistency', 'Decidability', 'Recursive functions' 'Truth'.

concept of **natural logic**. Its nearest conceptual environment on the side of logic itself is formed by the notion of *symbolic logic*, especially *predicate logic*, and that of *objectual reasoning* as opposed to *symbolic reasoning*. On the other hand, this concept is closely related to that of *cognitive rhetoric* as a theory founded on logic.[11]

The English term *logic* means not only the science of correct reasoning, defining, etc., but also a reasonable thinking, a good sense. Let the term *theoretical logic* be applied to the former, while the latter, being a quality or a conduct of mind, deserves the name of *natural logic*. In fact, there is a multitude of natural logics, as great as that of various natural languages, or even as that of human individuals; but even if there are so many of them, they have much in common, sufficiently much to form a vast field to be called 'natural logic' in the singular.

The field so called has to be subjected to some laws which, being concerned with behaviour, can be stated as certain rules. There then arises the question of how such rules of natural logic are related to the rules of theoretical logic, especially its standard symbolic version known as predicate logic. Are these sets of rules identical, or disjunct, or else overlapping? May natural logic profit from relations with theoretical logic, and vice versa?

To answer these questions, it should first be realized that there are two component parts of natural logic, a biological constituent and a cultural constituent, the latter being mainly linguistic.[12] Owing to the biological constituent, each of us is capable of **objectual** (material) inferences. That is to say that in the reasoning about an object one may come to the true conclusion without verbalizing either the premises or the conclusion. The reasoning consists then in a sequence of **mental transformations** of the object in question. It can be nicely shown for some geometrical objects, as

[11] The concepts of objectual and symbolic reasoning are first introduced in Chapter Two, Subsection 3.1, and are then discussed in the context of generalization procedure in Chapter Seven, Subsec. 3.2. The definition of cognitive rhetoric is found in Chapter One, Section 1.3 *in fine*.

[12] The biological constituent of natural logic is discussed in Chapter Seven, Subsec. 2.2, in relation to von Neumann's ideas.

did I. Kant in the example of the proof that the sum of the angles in any triangle equals two right angles.[13]

Once an objectual inference is verbalized as a sequence of sentences, it can be examined to search for logical rules to justify particular steps in that mental processing of the object. As far as mathematical reasonings are concerned, predicate logic proves sufficient to account for their validity. This means that, in mathematical domains, natural logic, as an *innate skill at objectual reasoning*, can handle a similar range of problems as that for which symbolic logic in its predicate version has been suited.

Certainly, in a case like that scrutinized by Kant, i.e., belonging to geometry, the biological constituent of natural logic is essential, because the intuition of space (to use Kant's category) is apt to guide not only human but also animal reasoning. On the other hand, we should not disregard the linguistic constituent such as the geometrical terminology; without it, the problem of the sum of the triangle angles could not have been raised at all. However, the linguistic constituent in geometrical inferences may lack logical terminology; it helps, but one may do without it.[14]

In this sense, the logic of reasoning like that mentioned above has been called natural — as one marking people from their birth, or acquired by them spontaneously without any effort to master it, without any study of logical theories. The question raised in the title of this Section is concerned with the natural logic so conceived. When it comes to its comparison with predicate logic, there comes the question of their relation to each other to be tackled in what follows.

3.2. There are two methods of extending predicate logic beyond that set of means which involves the categories of expressions and inference rules introduced so far. One of these methods consists in adding new axioms or new rules which are not derivable from

[13] This example is comprehensively discussed in Chapter Seven, Subsections 3.2, 3.3, 4.1.

[14] This observation is confined to reasonings concerning individual mathematical objects. When a reasoning is concerned with a whole mathematical theory, e.g., when its consistency is examined, then theoretical logic is necessary *ex definitione*, as a theory dealing with cognitive values of other deductive theories.

the existing ones; the other consists in defining new categories of expressions, with corresponding axioms or rules, with the help of already existing means. The latter does not increase the deductive power of predicate logic but makes it much more operative. Both methods deserve a careful study from the rhetorical point of view to confront the inferential means of predicate logic and those which people owe to their natural logic.

The first and most usual extension consists in adding axioms which allow new deductions, and at the same define the symbol of **identity** '=' as a new logical constant, the binary predicate added to the truth-functional connectives and the quantifiers.

It can be defined either by appropriate rules or by a set of axioms. Since the latter is an instructive example of what is called **implicit definition** (to be discussed later), it is advisable to formulate it as a set of axioms. It consists of two axioms from which other propositions characterizing identity can be deduced. The axioms are as follows.

$$\begin{array}{ll} x = x & reflexivity; \\ (x = y) \rightarrow (A(x) \rightarrow A(y)) & extensionality. \end{array}$$

The terms 'reflexivity' and 'extensionality' are names of properties defined by the respective formulas. Here are the other properties characteristic of the relation of identity, derivable from the above axioms.

$$\begin{array}{ll} (x = y) \rightarrow (y = x) & symmetry; \\ ((x = y) \land (y = z)) \rightarrow (x = z) & transitivity. \end{array}$$

The identity theory added to the predicate calculus makes it possible to introduce new individual names. The method consists in forming a name out of a predicate with the help of the quantifiers and the identity symbol. All propositions which can be expressed in this new form, i.e., involving names, are also capable of being stated in the old form, i.e., with the use of predicates alone. Although this extension does not advance the deductive power of predicate calculus, it is of utmost importance, because it enables the introduction of function symbols into the language of mathematics and thus ensures maximum efficiency in computing (see Subsec. 3.4 and 3.5 below).

Obviously, the relation of identity or, as it may also be called, equality, is not alien to natural logic. However, at the linguistic level of this logic the symbol '=' does not have an exact counterpart either in English or in similar languages. Expressions such as 'the same', 'identical with', etc., are not so convenient, so operative, in the syntactic aspect, while the verb 'is', which syntactically is most similar to '=', is burdened with an ambiguity; sometimes it means the same as '=', for instance in the context 'two and two is four', but is different in the sentence 'there is an even number'; in still other contexts 'is' should be interpreted as a counterpart of the inclusion sign of the theory of classes. The moral to this comparison is that we deal here with a case in which theoretical logic helps natural logic in clearing up an ambiguity, and so contributes to its enhancement.

3.3. Predicate logic can be developed without the theory of identity. However, there would be no point in such abstention since the predicate '=' is necessary for practical reasons in the language of mathematics, and there are no objections to be raised against it from a philosophical point of view. This is why the theory of identity is usually seen as an integral part of predicate logic.

There is another extension to increase the deductive power of predicate logic, one being neither so necesssary nor so undisputed, yet useful practically and interesting philosophically. It is the predicate calculus of second order or, shortly, **second-order logic**, from which the theory discussed so far is distinguished by the name of **first-order logic**.

The vocabulary of that first-order calculus, let it be recalled, contains, apart from logical constants, individual variables (possibly, individual constants, too) and predicates. The status of these predicates, symbolized in the foregoing exposition by capital letters, as 'P', 'Q', etc., was not discussed as yet. The present context gives us an opportunity to explain that such letters should be interpreted as predicate constants despite their being single letters, and not full-fledged expressions from a concrete vocabulary, say English. If we use letters in this role, it is for the sake of convenience; otherwise we would be bound to decide to which domain the predicate calculus is to be applied (and so to use predicates concerning the chosen domain) instead of drawing attention to its universal applicability. Hence, such a letter functions as a predicate supposed to have a definite meaning which, however, is irrelevant to the validity of reasonings (being our sole concern in developing the calculus), and is therefore disregarded by us.

In the second-order calculus, when we use single letters for the predicate category, we assign them a different role, namely that of predicate variables; let Greek capital letters be such variables. Consequently, we need predicates of a higher order to be predicated of such variables, that is to say, we need predicates to make sentential formulas out of such variables as their arguments. The appearance of that second level explains why the new calculus is called second-order logic. The process of adding new levels can be continued to obtain next orders of logic.

The addition of the second order of predicates has far-reaching consequences both in the technical and in the philosophical dimension. Technical advantages for mathematics are thoroughly discussed by Hilbert and Ackermann [1928] (a pioneering work in this field), also by Barwise [1977]; e.g., in arithmetic the induction principle can be conveniently stated in second-order predicate logic.

As for philosophy, there is, for example, the well-known second-order formalization of the Leibnizian principle of the *identity of indiscernibles* which runs as follows:

$$x = y \equiv \forall_{\Phi}(\Phi(x) \equiv \Phi(y)).$$

This formula opens a new prospect for the theory of identity, as it defines the predicate '=' in terms of equivalence and the general quantifier alone, without additional axioms. Its philosophical merits are obvious for those, say, who deal with the major problems involved in Leibnizian philosophy. Yet despite such a significance, it cannot be formalized in the first-order language. It should be noted, however, that one takes advantage of the second-order language provided that this language itself is not rejected by that person for philosophical reasons. Objections raised by some critics are connected with the rule of introducing the existential quantifier (see above, Subsec. 2.2) which in second-order logic amounts to acknowledging the existence of abstract entities, the point decidedly objected to by nominalists. However, whether we succeed in either refuting or ignoring such objections or not, we have to agree that for some arguments, especially in philosophy, as in problems related to the Leibnizian principle, the second-order logic is an extremely useful device.

The possibility of extending logic towards ever higher orders, i.e., categories of predicate variables, up to infinity, is an obvious advantage of predicate logic over natural logic. Only the means elaborated in symbolic logic are fit enough to render that infinite array of categories resulting in what is called the *predicate calculus of order omega*. It is akin to some versions of set theory which in its history proved to surpass the abilities of natural logic. This extreme extension of predicate logic should be of special use in philosophical argument concerned with infinity, a realm so alien to natural logic that it may get lost in those new surroundings.

On the other hand, even this far-reaching extension of logic towards a treatment of abstract entities is not sufficient for some simple arguments dealing with a category of abstract names. It is the category of **names of properties**. The abstract entities of higher-order logics are not those which incessantly appear in philosophy, in humanities, and in every-day discourses, namely properties attributed to individuals, and also to other properties; the latter deserve to be called properties of higher orders but this analogy does not throw a bridge between predicate logic and natural logic. This discrepancy is worth a careful study, and for the present purposes let the following example illustrate the problem.

Among the most famous philosophical arguments are those stated by Descartes in his *Discours de la méthode*. The sequence of arguments begins with the statement CM (Cartesian Maxim) to the effect:

(CM) *The good sense is a thing evenly distributed among humans*

(in the original *le bon sense*, and *bona ratio* in a Latin version).

Good sense is a property of individuals the possession of which by every individual can be rendered in first-order logic in the following form;

(CM*) $\forall_x S(x)$,

with the universe of humans and the predicate 'S' to abbreviate the phrase '... possesses good sense'. In second-order logic we can use a predicate having the predicate 'S' as its argument, but it is not what we need. What we need is the phrase 'good sense' alone to be used as the grammatical subject in a premise, and to denote that property of which another property is predicated, namely that of being distributed, and again of the latter property, namely distribution, it is predicated in the concise adverbial form that the distribution is even. Thus, something like natural logic of third-order is involved in such a short and simple statement, but it is not a logic likely to be rendered in a third-order predicate logic.

Since (CM*) is expressible in natural logic, e.g., *via* English (as a language whose logic is part of natural logic), we can ask about logical relations holding between them. Obviously, the asterisked maxim follows from the other but not *vice versa.* If good sense is evenly distributed among humans, then each human is endowed with it; but from the latter there does not follow the fact of *even* distribution as stated in (CM). Neither inference nor the lack of inference can be ascertained by the third-order predicate calculus (as the only candidate, if any, to be authorized to settle these questions from the standpoint of theoretical logic). Since in predicate logic there are no syntactic and semantic categories for properties, properties of properties, and so on, there are no rules to guide and control inferences involving these categories. Nevertheless, in English, in French, in Latin, etc., there must inhere such rules, if not explicitly stated, then at least acting in an implicit way, so that we can be certain of the validity of such inferences, as well as capable of stating *non sequitur*, that is the lack of logical following, if it is the case.

There is a means in predicate logic which makes it closer to natural logic as dealing with names of properties. It is the **abstraction operator**, i.e., one which transforms a sentential formula into a name of the class of those things which satisfy that formula; it does not bridge the gap in question but deserves to be mentioned in connexion with properties as abstract entities.[15]

[15] The abstraction operator plays a significant role in applications of logic, also to natural language, especially in its generalized form called lambda-operator. In spite of this role, which is of consequence from the rhetorical point of view, the discussion of this operator would exceed the limits of the present essay.

The formal definition suited for one-argument predicates, where the cap above the variable plays the role of the operator and square brackets mark its scope, runs as follows:

$$\forall_z(z \in \hat{x}[S(x)] \equiv S(z))$$

Let 'S' mean the same as above ('... possesses good sense'). This formula then means that any human individual z belongs to the class of those possessing good sense if and only if he possesses good sense. The constituents of this equivalence describe the same state with the same words but in a different syntactical manner. On the left side there occurs the name 'class of ...' which does not appear on the other side. Obviously, whatever can be said in one of these ways can also be said in the other, and in this sense the extension of the language by adding the abstraction operator is inessential. Yet, when combined with the rule of introducing the existential quantifier, this definition results in the statement about the existence of a set, as do statements of second-order logic, e.g., the set of humans evenly endowed with good sense. Now we can predicate a property about that set, e.g., that it is nonempty, that it contains more than three members, etc. However, this does not help natural logic as concerned with properties of individuals (such as good sense), properties of such properties, etc. Classes as well as properties are abstract entities, and they are related to each other in an important way, yet they constitute different categories of abstract objects, irreducible to one another. This contrast emphasizes some features characteristic of natural logic which are not reflected in theoretical logic.

These and other differences between the two logics require a diligent study in order to be explained and, hopefully, removed. Before such a study is undertaken, let it suffice to notice them to become aware that for rhetorical purposes we must go beyond

---

Some basic information on this subject is found in *Logic* [1981], esp. in the articles 'Combinatory logic' and 'Lambda-operator' by A. Grzegorczyk. A more advanced exposition of the lambda-calculus is given in Feys and Fitch [1969] and an instructive example of its linguistic applications is provided by Cresswell [1977].

theoretical logic, however sophisticated the array of ever-higher-order logics is.[16]

3.4. The extensions which are discussed below are inessential, as mentioned above, in the sense that when the extended logic is applied to a theory, the set of theorems derivable in it (due to that logic) is the same as in the case of applying non-extended logic. This is not to mean, though, that the extension is of little use. Some extensions are necessary for reasons of practicality, to develop mathematics, as is, e.g., the introduction of function symbols, while other ones improve the technical side of a theory and, moreover, prove inspiring in a philosophical aspect.

The first to be discussed, both for systematic and for historical reasons, are the theories of definite descriptions. Their beginnings go back to Frege [1893] and Russell [1905] but for present purposes it is enough to make use of the theory developed by Hilbert and Bernays [1934-39].[17] The theory devised by David Hilbert (the main author) is in accordance with the inferential (i.e., rule-oriented) approach adopted here in regard to quantifiers, and constitutes a useful introduction to the later discussion of definition (Chapter Eight, Subsec. 2.3 and 2.4). This is why it has been selected for the present purposes.

$$\frac{\begin{array}{c}\exists_x A(x)\\ \forall_x \forall_y (A(x) \land A(y) \to x = y)\end{array}}{A(\iota_x A(x))}.$$

For purposes of the present discussion it is more convenient to present the above rule in such a form as to indicate possible occurrences of free variables $z_1, \ldots, z_n$:

$$\begin{array}{ll}(1) & \exists_x A(z_1, \ldots, z_n, x)\\ (2) & \underline{\forall_x \forall_y (A(z_1, \ldots, z_n, x) \land A(z_1, \ldots, z_n, y) \to x = y)};\\ (3) & A(\iota_x A(z_1, \ldots, z_n, x)).\end{array}$$

[16] This reservation about the applicability of higher-order logics should be combined with attempts to take as much as possible from them for understanding and developing the logic of natural languages. Such an attempt is made by Gallin [1975].

[17] A review of various approaches as a historical introduction to the problem is found in the article 'Definite description' by W. Marciszewski in *Logic* [1981].

Formula (1) is called the **existence condition**, and formula (2), the **uniqueness condition**.

This rule makes it possible to eliminate the existential quantifier in those cases in which it has been proved that the object satisfying the formula A exists and there is no other object satisfying A. Then we are allowed to introduce the following **definite description**:

(4) $\iota_x A(z_1, \ldots, z_n, x)$,

and on the basis of (1) i (2) to obtain (3), and at the same time to define the function

(5) $f(z_1, \ldots, z_n) = \iota_x A(z_1, \ldots, z_n, x)$.

From (3) and (5) we can derive

(6) $A(z_1, \ldots, z_n, f(z_1, \ldots, z_n))$,

hence a formula in which the existential quantifier does not occur.

Thus the elimination of the existential quantifier is an operation which consists in omitting the quantifier and replacing a variable (formerly bound) by the individual constant defined by the given description.

However, we hardly have a definite description at hand when dropping the existential quantifier. Then we use a symbol, say 'a', to stand for any object which the predicate in question refers to, as prescribed by rule **EE** (cf. 2.2 above). It is not necessary for truth-preserving that a be unique, we content ourselves with its existence. Hence the rule which introduces 'a' to the language is like the formerly stated rule for definite descriptions, but with the difference that the uniqueness condition is omitted. An expression introduced to a language by so liberalized a rule is called **indefinite description**.

In those natural languages which possess articles, the definite article is what forms a name being the counterpart of a definite description, and the indefinite article forms what corresponds to an indefinite description. In a language which lacks articles, as Latin, the counterpart of indefinite description can be characterized in terms of *member supposition* as discussed in Chapter Four (Subsec. 3.1).

At the same time, a name which satisfies only the existence condition and is allowed to designate more than one object resembles the familiar general names of traditional logic; its semantic interpretation is as that of the non-empty predicate of which it has been formed. The treatment of general names as indefinite descriptions settles the question of translating universal statements of traditional logic into class logic and predicate logic. The problem was raised in the discussion of the theory of classes (Chapter Four, Subsec. 2.1 and 2.2) and there two methods of class-theoretical interpretation of the universal statements were presented, one of them called **strong interpretation**, the other called **weak interpretation**.[18]

Let us adopt the same distinction for universal statements rendered in predicate logic (UA means universal statement, the subscripts 'w' and 's' hint at the weak and the strong interpretation, respectively).

[UA_w] $\forall x(Ax \to Bx)$
[UA_s] $\forall x(Ax \to Bx) \wedge \exists x Ax$.

The weak interpretation consists in treating UA as a statement on non-existence. Thus, 'Every Cretan is a liar', means the same as 'There are no Cretans who are not liars'. This statement would remain true even if there were no Cretans at all, i.e., if the term 'Cretan' were empty.

That interpretation is confirmed by an analysis of UA_w. Let the above sentence be rendered as

[1] $\forall x(Cx \to Lx)$,

where respective letters are abbreviations for the predicates 'is Cretan' and 'is a Liar'). Now, using the rule EU (Elimination of Universal quantifier), we obtain:

[2] $Ca \to La$;

The next step consists in expressing the above conditional in the following form:

[3] $\neg(Ca \wedge \neg La)$;

[18] See, e.g., Leśniewski [1992], vol. 1, p. 377. Illuminating historical data as to existential import of universal propositions are found in Kneale and Kneale [1962], and in Simons [1992]. The latter also comments on this problem in Franz Brentano, recent free logics, and especially Leśniewski.

after applying the IU rule (Introduction of Universal quantifier, see 2.2 above), we again have a quantified expression, viz.:

[4] $\forall x \neg (Cx \wedge \neg Lx)$

to the effect that it is true of everybody that he is not both a Cretan and non-Liar, that is to say, that: there does not exist anybody who is a Cretan and is not a liar. The last transformation is due to some relation between the existential and the universal quantifier, not discussed till now but being so intuitive that it can be seen when comparing [4] and the following:

[5] $\neg \exists x (Cx \wedge \neg Lx)$.

The last formula demonstrates that from any universal affirmative statement there follows a negative existential statement, i.e., a statement about the non-existence of any object having such-and-such properties (here the properties of being a Cretan and of not-being a liar).

The relationship between predicate logic involving descriptions and the natural logic of articles (and similar constructions) requires a careful further study in two directions, the philosophical and the linguistic. The first can be exemplified by the extensive and thorough study by E. M. Barth [1974], the latter by a chapter in Hans Reichenbach's inspiring textbook [1948] trying to apply symbolic logic to natural language. Some authors look for still other ways to bring symbolic theoretical logic closer to natural languages. Peter Simons, for example, stated a program of modern theoretical logic in a way related to traditional logic and modernized according to S. Leśniewski's principles.[19]

3.5. The theory of descriptions throws a bridge between predicate logic and the concept of function belonging to key concepts of

[19] Simons [1992a] promises an introductory textbook to complete in some way Leśniewski's logic, which he sees as bridging the gap between traditional and modern logic. Cf. Simons [1992] (the chapters concerning Leśniewski's logic). An illuminating introduction to Leśniewski is due to G. Küng in *Logic* [1981] (see also Küng [1967]). As for Hilbert, whose approach is followed in this discussion, he did not develop the description theory towards applications to natural language, for he devised it as a step only in the proof-theoretical procedure of eliminating quantifiers.

mathematics.[20] Here is one of those points in which the great program of unifying mathematics and connecting it with logic, initiated by Frege and Russell, has been successfully accomplished.

A **function** f is a rule which assigns to each member x of a set X a unique element y from a set Y (not necessarily different from X). The set X is called the **domain** of f, the element y assigned to x is called the **value** of f at x. In other words, a function is a rule for setting up a correspondence.

To express a correspondence we need a two-place predicate; obviously, a description can be formed of a two-place predicate, provided it is, so to speak, a relational description. At the same time the requirement of uniqueness ensures, in each case of correspondence, the uniqueness of the value corresponding to each element. To adduce some natural-language counterpart to the transition from a predicate to a functional expression, let us consider the predicate 'is father of' used in the context

(i) *y is the Father of x*, abbreviated as $F(y, x)$.

Its functional counterpart is the name 'the father of' in the context

(ii) y = *the father of x*, abbreviated as $y = f(x)$.

In this example (ii) is a rather deviant representative of natural language since the equality symbol hardly belongs to the vocabulary of ordinary English, which at this place would provide us with the verb 'is'. This deviation is deliberately committed for two reasons, namely, to distinguish (i) from (ii), which otherwise would take exactly the same form as (i), and to exemplify how a natural language can be freed from some ambiguities with the help of theoretical logic.

The general method of making functions from descriptions is reported above (Subsec. 3.4) as the Hilbertian procedure of eliminating the quantifiers (see line (5)). Applying this method to the present example, we obtain:

(iii) $\iota_y F(y, x) = f(x)$,

where on the left side one puts the description formed of the predicate formula (i), and on the right side, the name formed by the

[20] This concept is also mentioned in Chapter Five, Subsec. 1.1 in connection with the idea of the truth-function.

function symbol 'f'. Since the left side denotes the unique y being the father of an x, it can be replaced by the single symbol y, and thus there holds: $y = f(x)$.

Despite the said ambiguity of the verb 'is', being either part of a predicate or a counterpart of the symbol '=', natural logic avails itself of the concept of function in the sense explained above, connected with definite descriptions. This is due to the instinctive method of identifying an individual by descriptions which can be successively added as the need arises; e.g., to tell a John Smith from other so named individuals, we ask about his father's name, which amounts to using a relational definite description; if it does not suffice, we ask about a temporal (the birth time) and a spatial (the birth place) relation, and so on. This leads to the conclusion that in this respect the predicate calculus with functions and natural logic remain in perfect agreement.

The theory of descriptions yields encouraging evidence that predicate logic and natural logic have much in common, and each of them can be better understood in the light of, or in contrast to, the other. Moreover, owing to predicate logic, natural logic becomes more conscious of its own laws and powers but also of its limitations, for instance those in dealing with infinity. On the other hand, natural logic challenges predicate logic by posing new questions, such as that of dealing with orders of properties. To handle all these problems is a task for a further inquiry which should bring predicate logic closer to natural logic, in some points as close as the old syllogistic was, and, on the other hand, make natural logic still more sophisticated.