CHAPTER SEVEN

Reasoning, Logic and Intelligence

1. Does a logical theory improve natural intelligence?

1.1. The outstanding philosopher Karl Popper once remarked that at the beginnings of human intellectual history it was the deeds of liars which incited a reflection on logic.[1]

> We may assume that the first (and almost human) function of descriptive language as a tool was to serve exclusively for *true* description, *true* reports. But then came the point when language could be used for lies, for "storytelling". This seems to me the decisive step, the step that made language truly descriptive and truly human. It led, I suggest, to storytelling of an explanatory kind, to myth making; to the critical scrutiny of reports and descriptions, and thus to science.

Each of the listed activities of the mind requires intelligence, be it a simple description, an inventive storytelling, or a critical scrutiny of either of them.

Among the theories created in the early stage of our civilization, there emerged one which set the task of the *critical scrutiny* of descriptions, explanations and arguments. We now call it **logic**. Its core is formed by the theory of valid reasoning, while **valid reasoning** means obeying rules which ensure the truth of the conclusion from true premisses; thus they control truth-preserving (*salva veritate*) transformations. Logic equips us with the codification of such prescriptions, called **inference rules**, and with a conceptual apparatus to deal with them. Thus logic was always regarded as a device to support our natural intelligence in its critical activities.

However, there was no point in speaking of natural intelligence before the rise of the research field concerned with **artificial intelligence**, abbreviated as **AI**. The AI theory is a branch of computer

[1] See Popper [1976], p. 189 f.

science which aims at understanding the nature of human and animal intelligence and specifically at creating machines capable of intelligent problem-solving. AI is closely connected with logic, which provides it with one of the main components of artificially intelligent systems (data-bases, heuristic rules, logical rules of inference).[2]

Owing to the rise of AI and owing to the fact that AI engineers are fully aware of the role of logic in their constructions, we may have a fresh look at the problem of how logic improves natural intelligence. Is it in the same manner which is applied in developing artificial intelligence (to endow it with more axioms, more rules, etc.), or some other way resulting, possibly, from insuperable differences between organisms and mechanisms?

The question cannot be avoided, should we take the intentions of the founders of logic seriously. As reported in Chapter Three, in both the Cartesian and the Leibnizian trends in logic learned authors contended that the study of logical precepts should improve natural human thinking; and that complied with the purpose of Aristotle himself. Also nowadays people happen to believe that one who is expert in logic should reason more efficiently than the rest of his neighbours; likewise, for instance, a person trained in mathematics surpasses the laymen in the ability of solving mathematical problems.

The line of reasoning which leads to this view is roughly as follows. Logic is the theory concerning most general methods of solving problems, that is the methods to be applied in any domain whatever, as are modes of reasoning, defining, etc. Provided that such a theoretical knowledge concerning practice improves that practice itself (as is the case in mathematics), it seems to follow that among persons having at their disposal the same factual premisses, the logician has advantage over the others in the skill of reaching conclusions. That skill, in turn, is characteristic of any

[2] The notion of natural intelligence has already entered the conceptual repertoire of AI students as a new technical term. See, e.g., Callataÿ [1992]. The phrase abbreviated as AI is construed either as the name of the field of research or the name of the subject matter of this research. To avoid confusion it is advisable to use upper case initials to refer to the former, and lower case letters to refer to the latter.

keen mind. Namely, to reach a conclusion means to solve a previously stated problem (as can be seen in detective stories). And the ability of efficient problem-solving constitutes the core of what we call **intelligence**. Ergo, a flawless and efficient reasoning belongs to that core.

When scrutinizing the validity of the above conclusion, I do not mean to challenge the prestige of logic or logicians. I see logic as an enormously significant factor in the development of our civilization; yet this is not to its advantage when it is expected to do things which it does not do, while its actual merits may happen to be overlooked. 'To take the bull by the horns', we should start from the fundamental distinction between *verbalized reasoning* and **unverbalized reasoning**. It is in the very nature of a logical theory that the forms of inference studied and codified by it are verbalized forms; hence the role of logic for the improvement of reasoning depends on how far verbalization is necessary for endowing reasoning with the required validity. To attack this problem, let us first take into account the skill of reasoning as found in mute animals.

1.2. There is a widely known case, that has already became a classic, of unverbalized problem solution, viz., that of Köhler's chimpanzee Sultan which fitted a bamboo stick into another, after many attempts to solve the problem of grasping fruit that was out of his reach. In spite of the whole distance between a human and an ape, a human would react in a similar way, as it is the only correct solution, and similarly he would not need any verbalized inference. The whole process of reasoning can be done silently in one's imagination; it consists in processing a mental image of two things, viz., the stick and the fruit. Before the agent fits a bamboo stick into another, he tries this strategy in a wordless *Gedanken-experiment* which leads to the hypothesis that with an extended stick one would overcome the distance to the fruit. This encourages one to externalize this imagined action in the form of overt, actual, behaviour. Here we deal with a doubtless case of what I have termed **objectual inference**, also called material, in contradistinction to **symbolic inference**, also called formal (Chapter Two, Subsec. 3.1).

The story shows that in some cases the use of words is not necessary for a reasoning to obtain an intelligent solution. However, one may argue that owing to a linguistic articulation of the problem and applying logical rules to it, the reasoning would be somehow enhanced. I intend to produce an example of such an articulation in order to examine the interaction of the objectual and the symbolic-logical component, and to estimate how much either contributes to the conclusion. This symbolic articulation will be of the kind called **formalization**, that is such that inference consists in processing symbols as geometrical forms, without any reference to their meanings, and each step in this process is explicitly legitimized by mentioning inference rules of the kind discussed in the preceding chapter (cf. Chapter Six, Subsec. 2.4).

Such formalized proofs, when compared with those occurring in ordinary practice in mathematics and other fields, are long and cumbersome, hence they do not occur as arguments in a discourse, yet they prove indispensable for two other purposes, namely (i) for the research concerning certain logical properties of deductive systems, such as consistency, completeness of inference rules, independence of axioms from each other, etc., and (ii) for the application of computers to reasoning. The computer, or rather the program which the computer is fed with, is either a **prover**, i.e., a software to prove theorems, or a **checker**, i.e., a software to check the correctness of a proof produced by a human. In either case the proof in question is formalized since the computer is capable only of processing physical objects (which are configurations of electric pulses), and not of processing their meanings (if attached to physical entities).

There is something remarkable in the case of checker as involved in the interaction between humans and computers. Namely, the proof to be checked should be fully formalized as required by the computer, and at the same time it should be comparable with proofs appearing in the human practice as far as its length and conspicuity is concerned (only then would the amount of effort put into producing the proof be surpassed by the advantages of automatic checking).

The logical examination of Sultan's reasoning should consist in its formalization; the difference between objectual and symbolic reasoning will then be most conspicuous, since formalization is the most perfect version of symbolization, i.e., verbalization in a symbolic language. To render Sultan's reasoning in a formalized way, I shall make use of a convenient method of formalization which has been devised as an interface between humans and computers in the process of automatically checking proofs.

2. The internal logical code in human bodies

2.1. The system to be used for the present purpose consists in combining a checker with a **many-sorted** system of logic, that is to say, a system which differs from the standard one presented in Chapter Six, by the fact that it introduces local universes of discourse, i.e., varying from proof to proof, and admits of as many universes as one needs in the proof in question.[3] This trick alleviates the burden characteristic of formalization because it makes formulas considerably shorter. For, once having defined the **sort** of objects to be discussed, one is not bound to introduce predicates for denoting the classes introduced as sorts.

The system to be used below is called **Mizar** MSE, the term 'Mizar' being its proper name (in a random way chosen after the name of a star), and the suffix 'MSE' being the abbreviation for Many-Sorted (predicate calculus with) Equality, that is a version of Mizar belonging to the most elementary ones (the other versions provide the user with functions, metalinguistic devices, reference apparatus, etc.). It should be noted that the formalized proof stated below is far from being typical of those usually produced in Mizar MSE, as the preparatory part, i.e., that which forms the section called 'environ' is unusually long when compared with the length of the proof itself. Usually, there is no point in producing such logically trivial demonstrations, but it is just that logical triviality which should be shown in the present discussion. This demonstration was tested by Mizar's checker and assessed by it as valid, hence the formalization performed is faultless according to the standard of the adopted system. The proof is recorded literally in the way required by Mizar MSE so that a reader interested in the

[3] As for many-sorted logics, see Barwise [1977a].

technical side could rewrite it and process the text in his computer, if previously equipped with the relevant software.[4]

Every Mizar MSE proof starts from the section called 'environ', which includes assumptions (each marked with a number followed by the colon), to be used and referred to in the course of demonstration, as well as the definition of the universes over which the listed individual variables should range (the operation is marked by the phrase 'reserve [variables] for [sort of objects]'). As mentioned above, the sort names do the task of some predicates which would otherwise be inserted into formulas at the cost of a considerable increase of their length. In what follows there are the sorts, or universes, described as agent, length (attached to a stick), etc. (if we carried out the categorization in another way, it might prove more adequate, but not as short as desired for the conspicuity of our example).

The thesis T to be proved is the instantiation of the consequent of the general law stated as (assumption) 1 in the form of a conditional (the prefixing phrase 'for ... holds' is the universal quantifier). Thus the remaining assumptions should state two kinds of facts: (i) the existence of instances of the predicates occurring in the antecedent of 1, and this is done in the lines starting from the operator 'given' (the individualizing operator applied to a sort of individuals); (ii) that the antecedent is satisfied by the given individuals, and this is being successively stated by assumptions 2, 3, and 4.

Mizar MSE surpasses other systems of computer-aided reasoning since it allows variants of proof to simulate various habits or preferences of humans proving theorems. This feature proves crucial for our discussion, as we will be able to compare two variants and then to discuss the question as to which of them, if any, is closer to the inside process of reasoning as performed by Sultan or his human associates.[5]

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[4] That software — created by Andrzej Trybulec — including the checker, the associated editor, etc., is freely distributed *via* e-mail (address *romat@plearn*, or *filomat@plearn*.

[5] The variants are identical in the part preceding the section entitled 'proof'; these identical parts are repeated to facilitate their rewriting, or copying, by a potential Mizar MSE user wishing to check them by himself.

Here are the meanings of the abbreviated predicates.

'D[n,t,z]' for 'z is the Distance between n and t';
'S[z,x,y]' for 'z is the Sum of x and y';
'L[z,s]' for 'z is the Length of s';
'R[n,t,s]' for 'n Reaches t using s'.

Here is the proof in both variants (the adopted typeface is to imitate the shape of letters as seen on the computer screen, and to clearly distinguish the Mizar MSE text from the surrounding context).

Variant A

```
environ
reserve n for agent;
reserve t for agentsobject;
reserve x, y, z for length;
reserve s for stick;
1:  for n,t,z,x,y,s holds (Dist[n,t,z] & S[z,x,y] & L[z,s]
implies R[n,t,s]);
given n' being agent;
given t' being agentsobject;
given x', y', z' being length;
given s' being stick;
2:  Dist[n',t',z'];
3:  S[z',x',y'];
4:  L[z',s'];
begin
C: R[n',t',s']
proof
5:  (Dist[n',t',z'] & S[z',x',y'] & L[z',s']
implies R[n',t',s']) by 1;
6:  R[n',t',s'] by 5, 2, 3, 4;
thus thesis by 6; end;
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Variant B

```
environ
reserve n for agent;
reserve t for agentsobject;
reserve x, y, z for length;
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reserve s for stick;
1:  for n,t,z,x,y,s holds (D[n,t,z] & S[z,x,y] & L[z,s]
implies R[n,t,s]);
given n’ being agent;
given t’ being agentsobject;
given x’, y’, z’ being length;
given s’ being stick;
2:  D[n’,t’,z’];
3:  S[z’,x’,y’];
4:  L[z’,s’];
begin
T: R[n’,t’,s’]
proof
thus thesis by 1, 2, 3, 4;
end;
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The difference between these variants consists in the number of derivation steps. Variant A, resembling more closely than B the usual method of formalization, includes two such steps. The first results in formula 5 by (reference to) assumption 1 by virtue of the rule EU (Elimination of the Universal quantifier, called also instantiation), and the second results in formula 6 by 5, 2, 3, 4 by virtue of the detachment rule (*ponendo ponens*) (cf. Chapter Five, Subsec. 4.4). In variant B there is only one derivation step in which instantiation and detachment merge into one operation. Which formalization is closer to the actual reasoning of an intelligent agent in the in question situation — this is a seminal question to be discussed below.

2.2. The problem of the adequacy of formalized reconstruction of a reasoning does not arise at the level of ordinary courses in logic or ordinary textbooks, even in the more advanced ones. They take it for granted that a single inference step is controlled by exactly one rule, as exemplified in variant A. The question only then arises when we deal with a reasoning mechanism whose operations are discrete — as are single derivations, each marked by a separate line in a written formalized proof — but, unlike in that proof, they are not necessarily programmed according to the standard set of inference rules. This means that a single transition from one state of a mechanism or automaton to its next state is not bound to correspond to a single logical rule.

The fact that variant B is also accepted by the Mizar MSE checker as a valid inference, hence that it is implementable by a computer equipped with that checker, demonstrates that there is a reasoning device (namely that devised by the Mizar MSE designer) which may correctly arrive at the same conclusion with the use of different algorithms of inference. This awareness could have arisen only after the meeting of logic with computers.

The main hero of the first such meeting was John von Neumann, the American mathematician of Hungarian origin and German training (in the Hilbert school), regarded as the father of the digital computer. His concise book *The Computer and the Brain* (1st edition [1957]) summed up the first phase of experiences concerning relations between logic, language, mathematics, the computer and the brain. He concluded with the following statements (p. 81 f. of the edition of 1979).

> It is only proper to realize that language is largely a historical accident. The basic human languages are traditionally transmitted to us in various forms, but their very multiplicity proves that there is nothing absolute and necessary about them. Just as languages like Greek or Sanscrit are historical facts and not absolute logical necessities, it is only reasonable to assume that logic and mathematics are similarly historical, accidental forms of expression. They may have essential variants, i.e., they may exist in other forms than the ones to which we are accustomed. Indeed, the nature of the central nervous system and of the message systems that it transmits indicate positively that this is so. [...] Thus logic and mathematics in the central nervous system, when viewed as languages, must structurally be essentially different from those languages to which our common experience refers.

Von Neumann's point expressed in the above statements is of utmost importance for logic, and specially for *logic viewed from the rhetorical point of view* proposed in this essay.

To emphasize this point I resorted to the trick of examining the supposed reasoning of an animal in terms of predicate logic in its Mizar MSE version. The fact that the reasoning is carried out by an ape does not prevent a generalization since a human placed in such a situation is supposed to behave in a similarly intelligent way (it is why we admire Sultan for his human-like performance), while the fact that a dumb creature is capable of such inferences settles the vital question: whether language is necessary for reasoning. The answer in the negative paves the way for the problem stated by von Neumann: if reasoning belongs to those processes which may occur either (i) without a linguistic counterpart or (ii) accompanied, or even supported, by verbalized inferences, then we should ask about the latter if the same logic controls both reasoning processes. Von Neumann conjectures the answer in the negative. If he is right, then in our arguments we should take into account both the logic involved in a language and that extralinguistic logic which is more the work of Nature than of Culture (though the latter may have some feedback influence, as in any interaction between biological and cultural domains).

The point claimed as fundamental for the present essay is identical with that suggested by von Neumann. In the preceding chapters as many logical theories have been presented as is necessary to state and to develop this key point. To express it conveniently, let me resort to the term **code**, endowed with so broad an extension as to cover both linguistic and non-linguistic systems. Two features are common to both of them: each is organized as a syntactic system governed by formation and transformation (processing) rules, and each of them imparts a structure to the processes of sending, transmitting and receiving signals between units of a system which it controls, be it a society, a nervous system, a computer set, or genetic machinery.

From the point of view of a human observer, a language used for communication can be said to be outside as functioning among the members of a society, while some processes which occur in human bodies, e.g., as impulses on the nerve axons, are said to take place inside them. Correspondingly, I shall use the terms **external code** and **internal code**. The notion of internal code can be regarded as a 'technological' alternative to what Fodor [1975] and other philosophers of artificial intelligence suggested to name the **language of thought**. Fodor argued that mental processes involve a medium of mental representation, and that this medium is like a language, e.g., thoughts are like sentences. The point of the present essay agrees with Fodor's representationism in acknowledging mental or neural counterparts of linguistic units, but there is the important difference which has been already stated in the comments pertaining to von Neumann's hypothesis.[6]

When applying the idea of internal code to processes of reasoning we again shall take advantage of the Mizar MSE reconstruction of Sultan's inference. In the described computer simulation there is a linguistic layer, namely the text put into memory and appearing on the screen, to which the physical processing of configurations of electric pulses corresponds. The former occurs in an outside code, viz., a language for communication between the computer and its user, while the latter occurs in a machine language, hence in an internal code.

Now we are able to articulate the Mizar MSE lesson: it clearly appears that inferences can be perfectly made in the code of a machine, be it a computer, be it a brain. Human inferences may have linguistic expression, and this is a fact of historic consequence; for a verbalized reasoning can be subjected to criticism from the outside, by the party to a dialogue, and this circumstance (as rightly emphasized by Karl Popper) is the one to which our civilization owes its enormous drive. However, with an inside code there can occur entirely silent reasonings, i.e., having no linguistic counterparts. This is why humans as reasoning animals share this ability with other animals and with some machines.

These facts throw light on the role of theoretical logic for the improvement of natural intelligence. Logical theories are hardly necessary to enable us to reason, even no language is necessary for that purpose, but both language and logic are unavoidable to examine reasonings with respect to the truth of their conclusions, and this includes examination of logical validity. This is what is meant by Popper in his text mentioned at the beginning of this Chapter. A logician, in spite of his education, may prove a less skilled reasoner, and thereby a less intelligent being, than, e.g., a great detective; yet, if we need to say why we praise such a detective we have to resort to the conceptual apparatus of a logical theory (which Sherlock Holmes also did when he wished to explain reasons of his successes); logicians are those who should be appreciated for providing us with such theories. It is by no means the

[6] An extensive discussion of the 'language of thought' hypothesis is found in Sterelny [1991].

whole merit of logical theories. Their equally significant contribution consists in providing us with an outside logical code. It may be much different from the internal logical code of a machine (in particular of an animal or human body) but having been created by humans it makes it possible to raise questions, make comparisons and try analogies which give researchers a chance of understanding the logic of internal codes.

The point developed in this Section to the effect that wordless reasonings play an essential role in the lives of humans and animals, has a peculiar feature which is worth noticing. On the one hand, this point appears evident in the light of the facts discussed here, on the other hand, it is vehemently attacked by the large camp of learned authors who have called themselves behaviourists. These deny the existence of any internal states of a biological machine, hence thinking must be regarded by them as silent speaking, the latter being an external behaviour. Reasoning is a kind of thinking, and therefore, according to the behaviourist doctrine, there does not exist any reasoning without a verbal expression.

The above conclusion is obviously wrong in the light of experiments like that of Köhler and also in the light of everyday observations of animals. Hence it would be a waste of time to engage in polemics in that matter, but the influence of the behaviourist doctrine upon some circles makes it reasonable to mention it in the context of this discussion. Let this point be addressed to the followers of behaviourism as the challenging question of how they interepret their view in the era of computers which must have internal states and operate according to an internal code because they have been so constructed by humans. As the constructors of reasoning machines, we know that such an internal code is necessary for a machine to perform inference operations. This is not the least important reason to believe in a logical code in human bodies.

3. The problem of generalization in the internal code

3.1. Both reconstructions of Sultan's inference offered in the preceding section may be criticised as too sophisticated as far as nonhuman reasoners are concerned. At the same time one may propose the more general argument that so simple a situation should also be handled with simpler logical means in the case of human reasoners. The point I am to vindicate is to the effect that to start from a general assumption, as in both variant A and in variant B (in the preceding Section), though not necessary for the validity of inference (see variant C below), gives us an insight into non-verbalized reasoning, and hence is a process of great significance for the task of influencing other minds.

To better state the problem, let me recall both the predicates employed in the formalized reconstruction and the general conditional stated with their help as the first assumption.
'D[n,t,z]' is to mean 'z is the Distance between n and t';
'S[z,x,y]' is to mean 'z is the Sum of x and y';
'L[z,s]' is to mean 'z is the Length of s';
'R[n,t,s]' is to mean 'n Reaches t using s'.
The assumption reads as follows:
1: for n,t,z,x,y,s holds (D[n,t,z] & S[z,x,y] & L[z,s] implies R[n,t,s]).

The objection which may be raised concerns the feature of universality characteristic of this statement. It may be argued that anybody who is to solve a problem like that of Sultan has to cope with a concrete situation in which there appears an individual fruit, and individual stick, and so on. Why should he generalize his observation in the form of such a universal conditional, that is a general law, and in a moment later descend from this universal statement to its concrete instance, describing just the situation he is dealing with from the very beginning?

To discuss this question, again let us take advantage of Mizar's flexibility in formalizing inferences. Using it, we can, as in a laboratory experiment, change a factor to watch its connection with other factors. Now let us change the set of assumptions (i.e., the content of the environ section). The use of the operator 'given', which is to create the names of concrete objects, is not delayed to the proof section, as was the case in variants A and B, but is made at the start, i.e., in the environ section. This corresponds to the fact of perceiving the said objects in the first moment of facing the problem by the reasoner. Then the assumed conditional referred to as 1, concerning the connection between the facts listed in the antecedent and that mentioned in the consequent, takes the form in which variables ranging over sorts (as in the former variants) are replaced by their substitutions, being like proper names.

The proof, so reformulated, runs as follows.

Variant C

```
environ
given n’ being agent;
given t’ being agentsobject;
given x’, y’, z’ being length;
given s’ being stick;
1:  Dist[n’,t’,z’] & S[z’,x’,y’] & L[z’,s’]
implies R[n’,t’,s’];
2:  Dist[n’,t’,z’];
3:  S[z’,x’,y’];
4:  L[z’,s’];
begin
C: R[n’,t’,s’]
proof
thus thesis by 1, 2, 3, 4;
end;
```

Also this variant is accepted by Mizar MSE.[7]

A comparative discussion of variants A, B and C is like a thought experiment in which we imagine how reasoning would run if we were placed in Sultan’s position. Mizar MSE makes us sure that the successive variants of verbalization comply with the requirements of logical validity as checked by the algorithm of formalization. Now it is up to us to decide which variant is closest to our thought process as imagined in this *Gedankenexperiment*. (An advantage

[7] For the sake of experiment let all the lines starting with ‘given’ be left behind at their former places (i.e. in the proof section, as in A and B). Then the system gives the error message which reads *sorry, 11 errors detected*, and when asked to list the errors it lets the user know: *unknown variable identifier*. The announced number of errors equals the number of the occurrences of the letters marked with the apostrophe in formula 1. The error consists in the fact that they were not previously declared as variables (including also indefinite constants), hence they are not recognized by the system. The operator ‘given’ introduces alphanumerical sequences (here single apostrophized letters) in the role of (indefinite) constants, and so authorizes their use in the proof, while the same role is played by ‘reserve’ for genuine variables.

of Mizar MSE over other systems of computer-aided reasoning consists in its ability to simulate *various* ways of deriving the same conclusion from the same assumptions, these ways being close to the mathematical practice, while in other checking programs each of them sticks to only one set of derivation rules as fixed in the system in question.)

When comparing variant B with A (as stated in the preceding Section), we encounter the question whether the transition from the aasumption to the conclusion takes two steps (as in A), namely instantiation and detachment, or only one, in which these two operations are, so to say, merged (as reconstructed in variant B). This question sheds light on a possible difference between the logic of our brain and the historically developed system known as first-order predicate logic, the difference conjectured by von Neumann (cf. this Chapter, Subsec. 2.2). In this case it may cross our mind that the difference consists simply in the simultaneity of mental operations as opposed to the sequential character of a formalized proof, in which each operation is recorded as a separate line. If this supposition is confirmed, it should have a considerable impact upon the ways of applying predicate logic to rhetorical practice. It may prove that an 'orderly' stated reasoning, one arranged in the manner resembling a formalized demonstration, is hardly understandable for someone endowed with a natural logical skill, because one's own mechanism is faster and more efficient.

A more involved question derives from the comparison of the present variant C, in which the assumption is a concrete conditional, with that feature shared by A and B which consists in assuming a general conditional. Which reconstruction is closer to the reality of the human brain or mind? This rather sophisticated issue is the subject matter of the next Section.

3.2. The answer to the question ending the preceding passage may seem obvious, even trivial. It would be to the effect that an adequate reconstruction involves the concrete conditional, for all the elements to be dealt with in problem-solving, such as a stick, etc., are individual entities, and those are the only ones mentioned in the conclusion. Hence, the argument would run, the act of generalization resulting in the universal sentence (about any fruit, any stick whatever etc.) is wholly superfluous; to climb to the top of generality and then immediately to climb down the plain of concreteness is no reasonable strategy if what one needs is to handle the concrete alone.

Yet let us look at the question at quite a different angle. True, when facing a problem like that of Sultan, I am not *bound* to employ the universal statement; but am I *able* to abstain from doing that? Let us also note a rhetorical side to the question. Among the most frequent errors in argumentation is that of **hasty generalization**, i.e., a generalization not supported by a suitable body of facts. But if the drive to generalize is so irrestible, we should rather encourage a *critical* generalization than blame people for having that drive.

Returning to the main point of the discussion, let us note that the question has an age-old tradition, and take advantage of some thoughts of our esteemed ancestors. That tradition revived in the work of the Dutch mathematician, logician and philosopher Evert Wilem Beth (1908-1964). Its most extensive treatment is found in the book by Beth and Piaget [1966].[8]

The book starts from the issue which Beth calls the Locke-Berkeley problem. I shall call it the **Locke-Kant problem** after the names of those philosophers who contributed most to the stating and discussing of the question (some authors used, in this context, to mention Berkeley as a strong opponent of Locke's position, but Berkeley's own answer was so vague that it can be disregarded in the present discussion). The history of the problem mainly involves Descartes, Locke and Kant. Each of them tried to solve a puzzle which, according to Beth, has been successfully solved by modern predicate logic as dealing with generalization and instantiation.

Descartes contributed to the issue with fitting comments on the nature of mathematical reasoning but without noticing the difficulty discovered later by Locke. Descartes' point is that *our mind is so constituted by nature that general propositions are formed of*

[8] That the book was written together with the famous Swiss psychologist concerned with logical thinking, Jean Piaget, proves Beth's interest in the borderline between logic and psychology and his anticipation of what is presently pursued under the name of cognitive science.

the knowledge of particulars. However, it is not enough to acknowledge that we cannot do without the knowledge of particulars. In the reasoning about a concrete object (e.g., a triangle) we must be able to reason about any object whatsoever in order to justify the generalization we attempt in our proof. It would appear — Beth comments — that, according to Descartes, it is the essence of the triangle, and not any triangle whatsoever, which is the object of the intuition.

It was Locke who asked what it is that justifies the transition from the particular to the general. In Beth's reformulation, more precise than the original statement, we have to do with two connected but different questions, namely:

(1) Why do we introduce into the demonstration of a universal mathematical proposition an intermediate phase which relates to a particular object?

(2) How can an argument which introduces an intermediate phase nevertheless give rise to a universal conclusion?

Locke's solution consists in introducing the idea of the **general object**, e.g., the general triangle. For instance, when demonstrating that *for any triangle the sum of the angles is equal to two right angles* we refer the conclusion to, as Locke puts it (cf. Beth [1970], p. 43):

> the general idea of triangle, ... for it must be neither oblique nor rectangle, neither equilateral, nor scalenon; but all and none of these at once.

In Beth's comment (quoted before the above excerpt from Locke) we find the phrase 'the idea of *general triangle*' while in Locke's original we read 'the general idea of triangle'. Obviously, these phrases are not equivalent, the latter is acceptable for empiricists and nominalists while the former (that in Beth's paraphrase) is not. However, it may serve as a convenient abbreviation. The situation will then be like that described by Twardowski in the following comment:[9]

> Vorerst sei noch bemerkt, dass wir behufs Vereinfachung des Ausdruckes statt von *Gegenständen der allgemeinen Vorstellungen* [...] von *allgemeinen Gegenständen* sprechen werden.

[9] Twardowski [1894], p. 106, italicized by W.M.

With the proviso that in what follows I shall use the phrase 'general object'; e.g., instead of saying that the perception (*Vorstellung*) of a stick employed to reach a fruit is general, the stick itself will be said to be general.

However, the observation that we have to do with the phenomenon of generality in human thinking does not solve the problem of the validity of generalizations. It was Immanuel Kant who suggested a thought-provoking solution which resulted from his theory of mathematical cognition as specifically differing from empirical and metaphysical cognition. It was Beth who duly appreciated Kant's contribution by quoting his interpretation of the Euclidean proof that the sum of the angles of a triangle is equal to two right angles (Beth praises Kant for describing the mathematical procedure in question 'in an arresting manner'). The use of symbolic letters in Euclid corresponds to Jaśkowski's notion of the indefinite constant and to some procedures of Gentzen and Beth. It expresses this remarkable combination of concreteness and generality which so attracted Descartes' and Kant's attention.[10]

The statement to be proved is to the effect: *The sum of the angles of a triangle is equal to two right angles.*

The proof starts from an expression like that *Let ABC be any triangle* or *Let ABC be an arbitrary triangle*, to express the generality of the object under consideration which is represented by a figure like this:

A E

B C D

Kant's reasoning runs as follows.

[10] See Beth [1970], Chapter IV entitled 'The Problem of Locke-Berkeley', p. 42 ff. The original form of the proof is found in *Elements*, Book I, Proposition 32; I. Kant, *Kritik der reinen Vernunft* [1781] a 716. The English version (quoted after Beth and Piaget [1966]) is Kant [1933], p. 579.

> Now let the geometrician take up these questions. He at once begins by constructing a triangle. Since he knows that the sum of two right angles is exactly equal to the sum of all the adjacent angles which can be constructed from a single point on a straight line, he prolongs one side of his triangle and obtains two adjacent angles, which together are equal to two right angles. He then divides the external angle by drawing a line parallel to the opposite side of the triangle, and observes that he has thus obtained an external adjacent angle which is equal to an internal angle – and so on. In this fashion, through a chain of inferences guided throughout by intuition, he arrives at a fully evident and universally valid solution of the problem.

It is remarkable that in Kant's exposition the proof does not contain any references to axioms or previously demonstrated theorems, otherwise than it was practised by Euclid, who arranged his proofs in a sequence starting from those deduced directly from axioms and postulates. Kant does not appeal to deduction but to a mathematical intuition concerned with spatial relations.

3.3. The above example is fit to do at least two jobs. It is to illustrate the main problem of this and the following sections, i.e., the issue of intelligent generalization. Apart from that main purpose, the above specimen of reasoning is perfectly suited to exemplify another point concerning human inferences, crucial for the theory of argument; let us mention it before resuming the main theme. It is the fact that the **skill of reasoning** much depends on its subject, that is to say, it may successfully handle a certain subject, and to fail when dealing with another. For instance, one may have a highly developed spatial intuition which makes him a perfect reasoner in geometrical deductions, and at the same time he may totally fail in reasonings concerning social life. It is so because our reasonings are usually **objectual**, and only exceptionally **symbolic**.[11]

This means that the success of one's reasoning mainly depends on one's familiarity with the object in question and one's skill in transforming that object mentally in the direction determined by the problem to be solved. Such skills are due to inborn abilities as

[11] These concepts were introduced in Chapter Two, Section 3, and mentioned in Three, Subsections 1.1, 3.4., the latter providing them with an important historical context.

well as training in the domain in question. This fact seems to be of little advantage to symbolic (i.e., formal) logic, as it is far from proving its necessity for reasoning and arguing. Yet, on the other hand, only a professional logician can detect such basic features of human intelligence, since the notion of objectual inference appears against the contrastive background of symbolic (formal) inference, and that can only be precisely defined in theoretical logic. The rhetorical moral to be drawn is to the effect that, in trying to win an audience over to our own position, we should first recognize not only the audience's beliefs but also their ability of objectual inference in the domain of objects to be handled in arguments; if we lack such a skill, we should either give up or, if possible, give the audience the necessary training.

Having so taken advantage of the example suggested by Kant, let us return to the issue of correct generalization, which is to be examined in this example, in accordance also with Kant's and Beth's intentions.

4. What intelligent generalization depends on

4.1. After having thus reviewed the historical output regarding the problem of general objects and generalization, we shall consider the solution starting from that of Kant, and at the same time taking advantage of modern logical tools (as suggested by Beth) and of the notion of internal logical code (as suggested by von Neumann). The text, as quoted below, in which Kant proposes his solution, should be read in the German original, otherwise we would miss the suggestiveness of the Kantian terminology.[12] Then I shall comment on it in the form of an English paraphrase.

> Die einzelne hingezeichnete Figur ist empirisch und dient gleichwohl, den Begriff unbeschadet seiner Allgemeinheit, auszudrücken, weil bei dieser empirischen Anschauung immer nur auf die Handlung der Konstruktion des Begriffs, welchem viele Bestimmungen, z.E. der Grösse, der Seiten und der Winkel, ganz gleichgültig sind, gesehen und also von diesen Verschiedenheiten, die den Begriff des Triangels nicht verändern, abstrahiert wird [...].
>
> Die Mathematik [...] eilt sogleich zur Anschauung, in welcher sie den Begriff *in concreto* betrachtet, aber doch nicht empirisch, sondern

[12] I. Kant, *Kritik der reinen Vernunft* [1781], A 713ff.

> bloss in einer solchen, die sie *a priori* darstellt, d.i. konstruiert hat, und in welcher dasjenige, was aus den allgemeinen Bedingungen der Konstruktion folgt, auch von dem Objekte des konstruierten Begriffs allgemein gelten muss.

The single figure drawn on a sheet of paper is something experiential, and yet it does the duty of expressing the concept of triangle without any loss of generality. It is so because in this experiential perception one takes into account only the operation of constructing the concept of triangle while disregarding those properties which are irrelevant to the concept under construction, i.e., those which do not alter this concept, as are, e.g., size, sides and angles.

Mathematics strives for intuitive perception (*Anschauung*) in which it deals with a concept *in concreto*. However, this concreteness does not amount to a sensory perception. It is just that perception which has been *a priori* introduced, that is constructed, by mathematics. Whatever results from the general construction postulates must universally hold also for the object of the concept so constructed.

The general postulates of construction (*allgemeine Bedingungen der Konstruktion* referred to by Kant) can best be exemplified by what Euclid calls 'requirements' and is usually rendered by 'postulates'. In Book One these are as follows.

(E1) A straight line may be drawn from any one point to any other point.
(E2) A terminated straight line may be produced to any length in a straight line.
(E3) A circle may be described from any centre, at any distance from that centre.

This interpretation of the Kantian term *Bedingungen* provides us with a fitting illustration of what may be called the intuitionistic approach, as is that of Descartes and Kant, in contradistinction to the logical approach to mathematical demonstrations. What in the former is seen as a certain ability given a priori to intuitively construct mathematical entities, should, according to the latter, be verbalized as a deductive system. These approaches are not bound to contradict each other, they may prove complementary in the sense that the intuitionistic approach pertains to the internal code while the logical one to the external code such as a historically shaped language of mathematics. There is a strong convergence between these codes; necessarily so, as there must be a feedback between them such that they influence each other. However, they are not identical and a significant difference is shown in the discussed problem of generality.

In the mental process as described by Kant, i.e., as a process somehow conditioned by an internal code, the concrete and the general constitute a single whole: the general is seen as if *through* the concrete. However, in a written text of mathematical demonstration, even when as close to intuitive perception as in *Elements*, these aspects are separated. In the above (footnote 10) mentioned proof of theorem 32 (unlike in the Kant's discussion of the same fact) the demonstration starts with a concrete triangle which is given the proper name ABC, and ends with a generalization in the form of the universal statement: if a side of *any* triangle be produced, etc.

However, it should be asked *by what right* we pass from a singular to a universal statement. The same step, when made in a reasoning concerning empirical objects, is blamed by deductive logic as the error of *non sequitur*, that is a lack of entailment; and even in more tolerant inductive logic a generalization procedure based on a single fact is regarded as wrong. Thus we return to Kant: only the presence of the general in the concrete justifies generalization. There is no mystery in such a merger if we only agree that what is successively formulated in the external code as a statement about a singular object and as an entailed statement about a general object, is, in the internal code, given simultaneously. Machines — and brains are machines too — are radically different from sheets of paper on which we write down our demonstrations; there are many simultaneous processes in a machine, while a sheet of paper is only capable of receiving sequences of symbols successively line by line.

4.2. In the above discussion of the Locke-Kant problem, the issue of generalization and general objects was restricted to mathematical concepts, in accordance with the intentions of those philosophers who raised and tried to solve it. However, the reasoning under study, that which enabled Sultan to solve his problem, is concerned with the empirical world. May we derive, then, any advantage from the discussion concerning the domain of mathematics?

Fortunately, it is mathematics which has most to do with the empirical world, and Sultan's problem is partly mathematical. The other discipline which provides us with ideas involved in the examined reasoning is praxeology which, as duly regarded by Ludwig von Mises, is like mathematics in its dealing with concepts given a priori.[13]

Both mathematics and praxeology appear at the level of animal thinking, therefore Sultan can act as the main character of our story. His reasoning lies within the limits of the capability of animal thinking carried out in an internal code; at the same time it can be expressed in a human language, hence in an external code, and can logically be formalized in a way capable of being checked by a computer. Having provided several variants of formalization, we are now able to ask which of them is closer to Sultan's supposed inference and which is more probably like that of humans, especially with regard to the issue of generalization. As a result of such a comparison, we should be better equipped to grasp the role of language for the validity of generalization.

Before we proceed to discuss the above question we should do more justice to the sophistication of Sultan's inference which was so far dealt in a much simplified manner. It was deliberately simplified to reveal the basic logical structure at the cost of some finer details, but we should not stop at so elementary a level. Once more, let me recall the main assumption (as occurring in variants 1 and 2) and the meanings of the predicates concerned.

'D[n,t,z]' is to mean 'z is the Distance between n and t';
'S[z,x,y]' is to mean 'z is the Sum of x and y';
'L[z,s]' is to mean 'z is the Length of s';
'R[n,t,s]' is to mean 'n Reaches t using s'.

The assumption reads as follows:

[13] This is Ludwig von Mises' [1949] idea endorsed by the present author. See Chapter Five, Subsec. 2.2, where the idea of the apriori character of action theory is briefly discussed.

1: for n,t,z,x,y,s holds (D[n,t,z] & S[z,x,y] & L[z,s] implies R[n,t,s]).

Assumption 1 involves two mathematical predicates D, and S, belonging to geometry (the latter interpreted as the sum of two sections), one predicate to coordinate the mathematical object *length* with the physical object *stick*, and one praxeological predicate R. This is a simplification, since no mention is made of the method of the physical carrying out of addition. The perception that such a physical operation is possible is what constitutes the creative element in the reasoning and substantiates assumption 1.

The following text, which adds necessary praxeological elements, should be a better approximation to the actual course of reasoning carried out in an internal code.

> If the distance between the point being the end of stick *one* and the goal point (i.e., the point to be reached, e.g., in order to grasp the desired fruit) equals the length of stick *two*, then the stick *three* whose length is the sum of the lengths *one* and *two* can be used to reach the goal and it can be made of sticks *one* and *two* through putting one of them into another.

The phrase (predicated of the extended stick) 'can be used to reach' expresses the idea of a means or a tool, hence another praxeological notion. The one formerly used was that of the goal, and this pair clearly exemplifies that kind of obviousness and unavoidability which is characteristic of basic mathematical notions. The statement that *to attain a goal one should devise means which do not exclude each other* is as obvious and necessary as that *a straight line may be drawn from any point to any other point*. Both yield, each in its own domain, general postulates of construction (*allgemeine Bedingungen der Konstruktion*, as Kant called them). For example, if one says 'You cannot have your cake and eat it, too', one applies the above praxeological postulate when interpreting the goal as a kind of happiness while the eating of a cake and the preserving of the same as means which exclude each other; wherefore — one concludes with geometry-like necessity — the success of such an action requires a choice between alternative means.

Owing to that likeness, mathematical and praxeological notions can be treated on an equal footing with respect to criteria of correct generalization. Thus the solution of the Locke-Kant problem, the issue primarily concerned with mathematical general objects, can also be adopted in solving praxeological ones; and those two kinds alone occur in Sultan's problem.[14]

5. The role of a theory for intelligent generalization

5.1 The tendency toward generalization is so fundamental and so irrestible even at the animal level that it should be seen as an instinctive drive. Therefore I shall term it **instinct for generalization**. It serves the instinct for survival since experience is necessary for an individual to survive, and generalization is the core of experience, even at the primitive level controlled by the laws of conditioned responses. E.g., a dog once hit with a stick tends to avoid other sticks, even those having different length, shape, etc., hence he must have acquired the general notion of a stick.

From that biological level we rise to the logical level when we note that the instinct for generalization is by no means infallible. There arises the question of the criteria of correctness, and once more it can be seen how errors contribute to the rise of logic; likewise, according to Popper, do lies (see the beginning of this Chapter).

One may object that because of this unreliability of generalization, the drive to generalize does not deserve the name of an instinct (as suggested above). However, I do not think that the feature of infallibility should be involved in the concept of instinct; those who think otherwise can in the present context treat the label 'instinct' as a convenient abbreviation which is to hint at an irrestibile force, actually involved in the animal tendency to generalizations. It is this force which must be seriously treated by logic; so far its textbooks warn of generalizations (unless supported by statistical methods). Yet in their everyday lives people as well as animals incessantly generalize (without any recourse to statistics),

[14] When claiming such similarity between mathematics and praxeology one should face the following question: why has mathematics developed into an omnipresent science whose results, owing to enormous chains of deduction, are farthest from being trivial, while praxeology lacks any age-old tradition, and its statements remain at the level of platitudes? The answer may be sought in the fact that human deeds (being the subject matter of praxeology) are hardly measurable objects, and that prevents non-trivial inferences.

and nevertheless they succeed to survive in spite of the warnings of logicians.

The strategy for logic which derives from the above stated assumptions is as follows. The logician should acknowledge the instinct for generalization as basically right, and then provide people with means to minimalize the risk of error (instead of discouraging them from generalizing at all). These means, though, cannot be restricted to statistical methods, as those are applicable only in special cases where one is able to make a measurement.

The answer to be accepted as at least as old as Karl Popper's critique of the neopositivistic program for science. There is only one method, which is as far from being satisfactory as is democracy in the matters of a political system, but like democracy is the only one which is feasible. This method consists in *creating theories to be confronted with facts of experience; the better a theory stands up to such confrontation the more it proves reliable.*

The discipline which most successfully rises to the occasion is mathematics (including formal logic). Contrary to the neopositivistic doctrine, there is no convincing reason to count mathematics as being radically different from empirical sciences; there are rather differences of degree, and mathematics is found at the top of the scale of reliability (the case of theoretical physics being so close to mathematics exemplifies that law of continuity).

So we come again to the problem of generalization. Mathematical general objects are so very general that they find a gigantic, one may say astronomical, number of applications in every domain of the universe, hence their existence proves to have been tested with the uttermost success. Let us take, for example, the success story of the number zero; we cannot live without zero, hence its existence is beyond any doubt (once upon a time, in the era of Roman numerals, people lived without it, but what a poor life it was!). It may be that mathematics has a certain advantage over the other branches of our knowledge which, possibly, derives from the fact that all animals, including humans, have enormous inborn capabilities of computing. Mathematics used by a small bee, or even a still smaller ant, is comparable with that functioning in huge computers. With animals other than humans, practical mathematics cannot be transformed into theoretical mathematics while with humans it can, and so they have a considerable initial capital to start with.

A similar benefit must be enjoyed by some concepts being at the bottom of other disciplines. For instance, linguistics as a theory of communication and praxeology as a theory of action may stem from innate practical abilities of communication and of action, respectively; out of such a rough material, fine and successful theoretical concepts may be made, even if their success yields to that of mathematics due to the lesser degree of measurability of the subject matter.

Thus the correctnes of a generalization should be judged by the degree of its success. Let the following example, again concerning the life of monkeys, explain the point.

> A chimpanzee was taught a specially devised sign language so that he could request an apple, a plum, or a banana. At the same time he learned to name some qualities of objects, as hard, soft, warm, cold, short, and long. Once the animal was given a nut. It was appreciated as very tasteful, but the monkey had no means of expression to communicate his request for more nuts. Then, one day, he found how to handle the problem. He asked for a *hard plum*.

Was it a right generalization? If suggested by a human botanist, it would certainly be wrong because of its inadequacy for botanic classification. But with regard to the monkey's purposes it should be accepted as flawless and, moreover, deserving to be admired for its linguistic ingenuity. Even if this story were hardly substantiated, it could be used as a fable to exemplify the postulated relativism in the assessment of generalizations, as for the present purposes the ascertainment of its credibility is not necessary.[15]

To conclude this part of discussion it should be said that the reliability of a generalization depends on the context of the theory

[15] Unfortunately, I cannot quote the original source of this story, as I found it reported in a daily newspaper without any references. Some psychologists with whom I discussed the case were sceptical about it, but I do not see reasons for scepticism. The demarcation line between a human and an animal mind will not be blurred until a monkey by himself suggests a *new sign* for nuts and defines it in terms of 'hard' and 'plum'. Only that would mean having the innate idea of a language, as humans are supposed to have, while the remarkable performance reported in the story may be explained by a combination of conditioned reflexes with the functioning of a logical gate as that for conjunction.

in which it is involved. If it fits well into the theory (it makes an effective use of the means of the theory, it increases the explanatory and predictive power, etc.), and the theory itself is well-confirmed (as is, specially, mathematics in its age-long history), then the proposed generalization deserves to be accepted. For instance, the monkey's theory that the new object (called 'nut' by humans) is a hard plum makes perhaps the best use of the means of the language at its disposal, and fittingly predicts the behaviour of human mentors. Therefore the general concepts of a plum and of a hard object prove fittingly formed, and the new generalization introducing the concept of a hard plum should be also assessed as correct — in spite of its deficiencies in the context of another theory, such as that developed by human botanists to cover an enormously wide range of facts. Thus the logical merits of generalizations, either deliberate or instinctive, should be judged by the cognitive usefulness of that theory to which they contribute.

5.2. In order to explain the notion of the **cognitive use of a theory** for the present purposes, I shall resort to a rhetorically relevant case study.[16] From a primitive example of generalization discussed above we should pass to a more sophisticated one. Let it be the concept of *European* as occurring in so many contexts which are specially relevant to exemplify the rhetorical approach to logic. This example is to show how much the assessement of defensibility of generalization is theory-dependent. In some circles it is claimed that the concept of European involves a reference to Christian values, in other circles no such claim is made, and in still others the same claim is contested.

[16] A more general explanation would need too much time; the problem has a long history related to the ideas of pragmatism in philosophy and methodology of sciences, hence it would require a historical discussion exceeding the intended limits of this essay. The strategy of resorting to case studies as the equivalent of a systematic exposition agrees with the point of this essay that the insight into essential features of an object should be provided by suitably chosen individual cases.

Let us examine some arguments for the position listed first. These will be taken from the article by Georges Hourdin, the founder of the Catholic weekly *La Vie*.[17]

> The argument starts from the realization that to be a European involves endorsing the tendency to limit national sovereignty in a certain way. Then the argument runs as follows: this tendency can be substantiated only by the Christian postulate of making peace combined with the Christian readiness for sacrifices, especially those made by nations for the sake of a more universal and peaceful society. The conclusion is to the effect that *to be a European involves endorsing the Christian postulate of making peace combined with the Christian readiness for sacrifices*. The author lists three great personalities, the founders of European unity, all of them having been inspired by these Christian ideas, namely the German Konrad Adenauer, the Italian (formerly the Austrian) Alcide de Gaspari, and the Lotharingian Frenchman Robert Schuman.

The mentioning of these three personalities adds a body of historical facts to Hourdin's theory of being a European. These facts should have been predicted by the theory, hence their being the case confirms it to some extent. That a prediction comes true does not amount to the definite verification of a theory, yet this increases the defensibility of the generalization involved since the theory proves cognitively useful.

However, in order to judge the cognitive use of a theory we need more than the existence of confirming instances. The theory should stand up the test of denying instances, conveniently called **counterexamples**. If such instances are listed, the theory may be defended by their refutation, or by their reinterpretation, or by proving their irrelevance, or by a modification of the theory itself, nevertheless an action should be taken to further the discussion.

As for the case under study, the statement to be tested (italicized in the above indented passage) should be interepreted as a universal statement to the effect: *Every European is one endorsing the Christian postulates* [etc.]. In other words, Christianity is a necessary condition of Europeanism. To check this proposition by search for counterexamples, we need a more precise definition what it means to be a European. It is the defender of the thesis

[17] The article was published in *La Vie* in September 1992; I read it in a translation which failed to give the exact date.

who should cope with this task. Suppose that he offers only a partial definition (as a complete one is indeed difficult) to the effect that those eminent politicians who contributed to the foundations of a united Europe were certainly Europeans.

This answer makes it possible to look for counterexamples. As far as the opponent knows, Winston Churchill and Charles de Gaulle were eminent politicians who considerably contributed to establishing a united Europe. Did they endorse the said Christian postulates? It is now up to the defender either to prove the Christian affiliation of the said persons or to withdraw, or else modify, his claim — unless he calls in question the relevance of the given counterexamples (which may on occasion be reasonable, too, but should be hardly expected in this case).

Should the defender succeed in counting Churchill and de Gaulle as Christian-inspired politicians, then the defended theory would score more points as far as its cognitive use is concerned. Since the main thesis of this theory is identical with the tested generalization, the latter proves more defensible than it had been before applying the testing procedures.

Any case study, though enjoying the merit of inspiring concreteness, has a darker side which consists in its incompleteness, and that may lead to a misleading one-sided picture. It is up to the author of such a study to minimize this drawback through a well-considered choice of the instances to be examined. I tried to do my best when hinting both at the phase of confirmation and the phase of attempted refutation, and also at the role of definitions and the role of a body of facts. If I still failed, then a critique of that failure should advance the understanding of the logic of generalization.

6. Logic and geography of mind: mental kinds of reasoning

6.1. Logic is able to greatly contribute to the geography of the mind and that, in turn, helps **cognitive rhetoric**, i.e., rhetoric addressed to an intelligent and benevolent audience (cf. One, Subsec. 1.3.). Note that the contention of this essay is that rhetorical activity needs logic most as (i) the foundation for a descriptive theory of mind, also as (ii) a language to express a critique of arguments, and least as (iii) a means to improve abilities to reason. Though logic is perfectly fit for the third task as well, there is no urgent demand for such services on the side of rhetoric, for natural intelligence proves sufficient to correctly make everyday inferences. On the other hand, these difficulties occur in the mutual understanding of arguments which seem to be insuperable, which encourages us to learn more about human minds, more that can be expected either from folk psychology or from academic psychology. Then one requires stronger cognitive methods, and these may come from logic.[18]

One of the reasons to substantiate this view is that logic provides us with ideal types or reasoning, defining, etc., which enable the use of ordering relation in the set of mental acts. For instance, one uses several examples to explain the meaning of a term in the course of an argument. Such a behaviour is disapproved of by **folk logic** which in every case demands complete and precise definitions.[19] Modern theoretical logic, though, is not so rigorous; it helps the other side to understand, e.g., the partner's instinctive tendency to use examples as approximating the ideal of definition in the degree necessary for the argument in question; at the same time logic provides the parties to the discussion with means of assessing whether such an approximation is actually relevant to the point being defended. (Other advantages which the philosophy of the mind may take over from theoretical logic will be mentioned below in this Section.)

6.2. A philosophical map of the mind owes much to the concept of a **formalized proof**. At least four zones of the mind concerned

[18] To emphasize this point, let me mention an argument between Dr. Karol Wojtyła (at that remote time the future Pope was a lecturer of ethics at Lublin Catholic University) and myself concerning the relation between logic and ethics. He defended the view that ethics gives more insight into the human mind than psychology does. I shared his remarkable, even paradoxical, point that a normative approach may grant us more descriptive knowledge than a descriptive one does, yet I decidedly preferred logic to ethics in that role.

[19] What I call *folk logic* (imitating the already existing concept of folk psychology) is to great extent shaped by entries found in general dictionaries and popular encyclopedias which in a simplified manner reflect the state of logic in the first half of the 19th century.

with reasoning can be distinguished through using this concept as a frame of reference. The exposition of predicate logic in this essay aimed at this concept as one of its main objectives, while the three-variant formalization of Sultan's reasoning carried out in this Chapter provides examples of how formalization can help in the hypothetical reconstruction of mental processes.

The notion of formalized proof enables us to use an ingenious trick which may be called **physicalization of logic**. For this purpose we should reserve the term 'proof' for something as visible and tangible as physical objects are, while the word 'reasoning' would be reserved for a mental process capable of being recorded and expressed by a proof. Thus a proof is a physical object, e.g., a sequence of three-dimensional (though perceived rather as two-dimensional) signs made out of dried ink, or a record on a magnetic tape, etc. It is just this trick which accounts for the fact that proofs can be handled by computers, and that people can think of developing this capability even towards artificial intelligence. Logical correctness of a proof is defined by a set of inference rules (such as those discussed in Chapter Six, Section 2); what is crucial about such rules is the fact that the transformations which they define are physical transformations of shapes of symbols without any reference to their meanings.

Owing to such physical concreteness, we have a solid basis to define a set of notions in terms of formalization procedure. First, as suggested above, let reasoning recorded as a formalized proof be called **formalized reasoning**. Now we are able to define the concept of **intuitive reasoning** as one which is (i) non-formalized but is (ii) acceptable according to certain standards maintained by experts, in particular mathematicians.

This reference to standards of mathematical intuition agrees with von Neumann's previously mentioned view on the historical relativity of logic and mathematics. It may still be better understood in the context of such theories as that expressed by Wilder [1981], e.g., in the following statement. "[The concept of] 'proof' in mathematics is a culturally determined, relative matter. What constitutes proof for one generation, fails to meet the standards of the next or some later generation. Yet the mathematical culture of each generation possesses generally accepted standards for proof. At any given time, there exist *cultural norms* for what constitutes an acceptable proof in mathematics." (p. 40, Sec. 10 'The relativity of mathematical rigor').

The adjective 'intuitive' was unnecessary before the emergence of the notion of formalized proof or formalized reasoning; previously it meant simply a proof, or reasoning, without any additional feature. Once the concept of intuition in reasoning emerged together with that adjective, a new mental space was discovered and opened to inquiry. Now being aware that a formalized proof deals with symbols (as physical objects) alone, we become able to pose the question of what intuitive reasoning deals with. We begin to realize that we must deal with objects themselves, namely those objects for which respective symbols stand for. But if we deal with objects themselves, somehow being presented to our minds, may we not sometimes (i.e., in some points of a proof) do without symbols at all? Is it not so that we need symbols and sentences only as steps of a ladder which even if placed differently, or in a lesser number, would equally enable to reach the top? The answer in the affirmative is obvious to those who practise proving theorems. It is the answer like that: we cannot do without any ladder at all, but the same result can be achieved with different ladders, i.e., with different wording, greater or lesser gaps in wording left to be filled up by a reader, etc.

Owing to such a reflection, the present author felt authorized to introduce the notion of objectual reasoning (Chapter Two, Section 3) in order to hint at this aspect of intuitive reasoning, which consists in its dealing with objects instead of with symbols. Thus, 'objectual resoning' stands for the same class as does the term 'intuitive reasoning', and analogous equivalence holds for the terms 'symbolic reasoning' and 'formalized reasoning'; the former member of each opposites hints at the matter of transformations (objects vs. symbols), while the latter at the way of processing (intuitive, or mental, vs. physical).

The awareness that there exist mental objects of reasoning which are representations of some extramental objects (physical or abstract ones), which we owe to the logical theory of formalization, proves crucial from the rhetorical point of view. Now one can realize that the addressee of his argument may mentally live in a world of objects very different from that of his own — in spite of speaking the same ordinary language, common to both sides. Hence a failure of one's argument may be a hint that first of all the partner's mental world should be recognized, and only then is the time ripe to look for convincing arguments, relevant to the results of such recognition.

The next mental domain which we can explore owing to the concept of formalized proof is that of **instinctive reasoning** as described above in the example of Sultan's logical performances. Again, a significant rhetorical moral should be drawn from the theory built around this concept. Note that in the case of a disagreement occurring in an intuitive reasoning, its sources can be investigated through an exchange of messages concerning the conceptual world of each party to the dialogue. Yet a disagreement whose sources go to the deep stratum of instinctive reasoning cannot be detected in a similar way, because the reasoner himself is not aware of the course and premises of his reasoning; this is why he cannot contribute to the mutual understanding between him and his partner. Hence a partner intent upon understanding the other one must possess more logico-psychological skill and devote more time to investigations in order to decipher the subconscious reasoning of the other side than is necessary in the case of conscious intuitive inferences.

An example of such deciphering was given above in studying Sultan's case. The adopted method consisted in several tentative reconstructions of the supposed process of reasoning, and that, due to a precise formalization, revealed several possibilities of valid inference (presented as variants A, B and C). When facing the choice between the variant including generalization and that lacking generalization, we resorted to the more general conjecture that there is an instinct for generalization which is part of the instinct for survival (such as the generalization involved in learning). Hence generalization is expected to appear in any perception, also that providing the assumption of scrutinized inference. Such theoretical constructions are unavoidable where no introspective or other experiential data can be used. Obviously, their result is only hypothetical but such hypotheses, when subjected to suitable criticism, lead to the next steps of research, and so advance our understanding of the domain under investigation.

Thus we have listed three ways of reasoning, all of them being relevant to the rhetorical point of view, namely instinctive, intuitive, and formalized reasoning. The first is developed without any words or symbols; in the second a wording is necessary and essential but does not match the whole content of reasoning; in the third, the wording is adequate for the content, so that one can check its validity without any regard to the meanings of the symbols involved.

It should be noted that in the last case the symbols are not devoid of meaning; but even if the meanings are disregarded, the validity of reasoning or its lack can be stated by a purely mechanical check, i.e., one taking into account only physical transformations of symbols controlled by specially devised inference rules. Thus formalized reasoning has, so to speak, two faces, one turned towards the human ability of understanding meanings, the other towards a mechanical device to check validity by tracing physical transformations.

The picture of two faces leads to the question of what will happen if the human-oriented face disappears and there remains only the machine-oriented one. The kind of inference which one deals with in such a situation is called **formal reasoning**. There is a difference and a similarity between formal and formalized reasoning. A formalized reasoning is one which has, so to speak, been given a feature of formality without losing the feature of having a meaning, while a formal reasoning is deprived of the latter. Thus the next point to be considered is formal reasoning, an issue related to the problem of artificial intelligence.

7. Formal ('blind') reasoning and artificial intelligence

7.1. Nowadays logic proves able to contribute to what people used to call **artificial intelligence** or **AI** for short (in a way predicted by Leibniz — see Chapter Three, Subsec. 3.3). The AI theory is a branch of computer science which aims at understanding the nature of human and animal intelligence and specifically at creating machines capable of intelligent problem-solving. AI should work in a way similar to the following.

The computer may be given a **data base**, i.e., a systematically organized store of relevant facts (fed by a human designer) equipped with retrieval facilities, or it may have to learn some of these facts. The machine is to generalize and compare, discover relations, and predict possible outcomes of actions. To solve such tasks the machine needs **heuristic procedures** — like those used by humans in searching for unknown goals according to some known criterion (as discussed, e.g., by Polya [1971]) — as well as **inference rules** supplied by logical calculi. It is claimed by AI theorists that in both points there is a considerable analogy between the reasoning of a human being and the reasoning of a machine. Is that claim right? This is the crucial question to be settled in dealing with the problem of import of logical calculi for *rational cognition* and *rational communication*, the latter presupposing the former, and both including the factor of critical arguing, so much stressed by Popper.[20]

Two alternative strategies of simulating human reasoning should be considered in AI research. Success or failure in this enterprise should provide us with evidence to help the understanding of human reasoning processes. The strategies in question are related to what formerly, viz., in Chapter Two, Section 3, was discussed as the opposition between objectual, or material, and symbolic, or formal, inference (i.e., reasoning); by virtue of this distinction material inference may also be called **informal**.

In what follows, first the notion of objectual inference will be examined in some examples and comments, then it will be compared with what we know about formal inference due to predicate logic. Both kinds of reasoning will be discussed with regard both to natural and to artificial intelligence.

7.2. The idea of artificial reasoning had been anticipated long before the computer became able to materialize it in a physical shape. The most famous forerunners were Hobbes and Leibniz. The latter invented an ingenious metaphor of **blind thinking** to render

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[20] There is a vast multitude of consequential problems of communication which cannot be even mentioned in this book, deliberately confined to the issues of the nature of formal and informal, conscious and unconscious reasoning as applied in an argumentative discourse. Fortunately, the reader may consult the penetrative essay on rational discourse by Posner [1982]; e.g., its analysis of the use of sentence connectives in natural language fittingly completes what has been said on this subject in the preceding chapter of this essay.

the nature of what nowadays we call formal reasoning.[21] A recent counterpart of that metaphor can be found in John R. Searle's [1980] much discussed story of a Chinese room which describes a *Gedankenexperiment*. It makes sense to recall both stories so that the idea of formal inference could appeal to our imagination.

Leibniz's concept of blind thinking may better be explained in the context of his point regarding the material side of mental processes. If the brain — he claimed — were blown up to the size of factory, so that we could stroll through it, we should then see the content of thoughts. When commenting on this point in terms of modern science, one might say something like that. Should we know enough (as we do not yet) about the neural mechanism, we would be actually capable of recognizing thoughts produced by that mechanism — as if someone placed inside a huge mechanical calculator would be able to read results of arithmetical operations recorded in positions of rotating cogwheels, configurations of cogs, etc. Numbers can be read from such mechanical states with no worse result than from inscriptions on a sheet of paper. Since linguistic units can be arithmetized, that is, given a numerical representation (as is evident in the functioning of computers), those inferences which are not about numbers can also be expressed as sequences of digits; a look at those sequences shaped as physical states should then reveal their numerical meaning (as five fingers may mean the number five) and thus reveal the non-numerical meaning coded in them.

There is no materialistic extremism in Leibniz's approach, there is just the awareness that one can coordinate physical objects with abstract objects, and so identify the latter on the basis of the former. Similar coordination — it is assumed — must have been done by Nature in human and animal bodies, and it is how the story of the visit to the brain should be construed.

Let us imagine now that the process of transformation of numerical data is so involved that the observer placed inside a brain or a computer is not able to follow the corresponding transformations of abstract objects, for instance, meanings of expressions.

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[21] The discussion of this notion in the context of the 17th century ideas is found in Chapter Three, Subsec. 3.2.

However, he notices the input and the output, and being sure that the processing mechanism complies with relevant criteria, such as rules of arithmetical operations, logical rules of inference, etc., he can rely on the final read-out even if he is not capable of checking the correctness of any of the intermediate steps.

Now imagine that the person in question is not an observer of a brain or a computer but its user. Then he uses his device to find the final solution without being engaged in approaching it himself and step by step. Thinking based on the belief in the reliability of the final result produced by the symbol-processing device is, so to say, **blind thinking**, hence Leibniz used to call it *caeca cogitatio*. To put it another way, the contact of the mind with objects discussed is not direct, but takes place through such signs as those instruments of thinking which represent objects assigned to them. In this mode of thinking the thing itself is not present in the person's mind; operations on large numbers are a simple example of this. As long as we remain in the sphere of small numbers, for instance when multiplying three by two, we still can be guided by some image of the object itself; e.g., we imagine an arbitrary but fixed triple of things and join to it one triple more. Such an operation can be performed physically or mentally even if we do not have at our disposal symbols of the numerical system. It is otherwise when we have to multiply numbers of a dozen or so digits each. In such a case we are deprived of that visual contact with them and have to rely on sequences of figures which represent them. Such sequences are physical objects assigned to numbers as abstract objects, and operations on figures are unambiguously assigned to the corresponding operations on numbers. For instance, juxtaposition, that is writing the symbols '2', '·', '3' one after another, is an operation on signs, and the corresponding operation on numbers consists in multiplying two by three.

Since objects themselves are not 'seen' by the mind in this mode of computing, and the mind's attention is focussed on their symbolic representations, the phrase 'blind computing' fittingly describes the situation. Leibniz believed that other mental operations, especially reasoning, can also be performed on symbolic representations of their objects, and thus one would deal with 'blind thinking', in particular, 'blind reasoning'. This is why he so intensely tried to create a logical calculus which could be handled by a logical machine, analogously to the arithmetical calculus successfully handled by his arithmetical machine.

7.3. Though no modern author applied Leibniz's metaphor to ariificial intelligence, it nicely fits into the present state of AI research (the adjective 'present' is a concession to those writers who claim that, in the future, computer thinking should be indistinguishable from that of humans). This state is adequately reflected in Searle's [1980] thought experiment mentioned above, which was meant to challenge the following claims of **strong AI** ('strong' means able to match human abilities):

1. that the machine can literally be said to *understand* a story told by a human which is demonstrated by correct and non-trivial conclusions drawn by the machine from that story, and
2. that what the machine and its program do *explains* the human ability of understanding the story, as displayed by drawing conclusions.

In the following story Searle attempts to show that these claims are far from being substantiated.

> Suppose that I am locked in a room and given a large batch of Chinese writing. Suppose furthermore (as is indeed the case) that I know no Chinese, either written or spoken, and that I'm not even confident that I could recognize Chinese writing as Chinese writing distinct from, say, Japanese writing or meaningless squiggles. Now suppose further that after this first batch of Chinese writing I am given a second batch of Chinese script together with a set of rules correlating the second batch with the first batch. The rules are in English, and I understand these rules as well as any other native speaker in English. They enable me to correlate one set of formal symbols with another set of formal symbols, and *all that 'formal' means here is that I can identify symbols entirely by their shapes.* (Italics by W.M., cf. Subsec. 7.2 above.) Now suppose also that I am given a third batch of Chinese symbols together with some instructions, again in English, that enable me to correlate elements of this third batch with the first two batches, and these rules instruct me how to give back certain Chinese symbols with certain sorts of shapes in response to certain sorts of shapes given to me in the third batch. Unknown to me, the people who are giving me all these symbols call the first batch 'script', they call the second batch a 'story', and they call the third batch 'questions'. Furthermore,

> they call the symbols I give them back in response to the third batch 'answers to the questions', and the set of rules in English that they gave me, they call 'the program'. Now just to complicate the story a little, imagine that these people also give me stories in English, which I understand, and then they ask me questions in English about these stories, and I give them back answers in English. Suppose also that after a while I get so good at following the instructions for manipulating the Chinese symbols and the programmers get so good at writing the programs that from the external point of view — that is, from the point of view of somebody outside the room in which I am locked — my answers to the questions are absolutely indistinguishable from those of native Chinese speakers. Nobody just looking at my answers can tell that I don't speak a word of Chinese. Let us also suppose that my answers to the English questions are, as they no doubt would be, indistinguishable from those of other native English speakers, for the simple reason that I am a native English speaker. From the external point of view — from the point of view of somebody reading my 'answers' — the answers to the Chinese questions and the English questions are equally good. But in the Chinese case, unlike the English case, I produce the answers by manipulating uninterpreted formal symbols. As far as the Chinese is concerned, I simply behave like a computer; I perform computational operations of formally specified elements. For the purpose of the Chinese, I am simply an instantiation of the computer program.

What Searle refers to as 'answers to the questions' can be identified with conclusions drawn from the text according to appropriate processing rules. For example, in another story told to exemplify such procedures it is said of a man that he ordered a hamburger, was very pleased with it, and as he left the restaurant he gave the waitress a large tip. Then the listener of the story (it may be a computer as well) is asked the question 'Did the man eat the hamburger?', and if he answers 'yes' it means that he draws a right conclusion from the set of data which contains those reported in the story and some other ones stored in the listener's memory as the knowledge about connections between some kinds of situations — such as the fact that a man who is pleased with his food is likely to consume it; logical inference rules to be applied to such data are the same as those appearing in Sultan's reasoning, variant A, viz., instantiation and detachment.

Thus the Chinese room story illustrates the nature of formal reasoning as one being carried out without any resort to the meaning of the text in question. This story not only contributes to our realizing that there are four kinds of mental behaviour in the process of reasoning (instinctive, intuitive, formalized, and formal), and so provides us with a mind-logical map for rhetorical purposes, but has also a direct rhetorical moral. Is it not the case that some people in particular situations behave like the inhabitant of the Chinese room, even when their native language comes into play? I mean repeating some slogans and applying correct inference rules to them, yet without understanding either them or their consequences.

Suppose that the slogans in question read: 'Laputians are wicked', 'every red-haired is a Laputian', 'the wicked should be punished'. Then one easily jumps to the conclusion 'all red-haired should be punished' even without understanding all the terms involved in the syllogism. In fact, at least some terms must be understood, for instance, in order to be able to identify red-haired and to inflict the punishment on red-haired people. However, there may be gaps in understanding, and these do not invalidate the formal correctness of reasoning.

Such a thoughtless reasoning is not purely formal, but the more it approximates a formal one (as an extreme), the more it tolerates gaps in understanding meanings, and in this sense the concept of formal reasoning contributes to explaining some forms of mental conduct encountered in society. Obviously, it is not **cognitive rhetoric** which may take advantage of such phenomena but rather that which deserves to be called **demagogic rhetoric**. Thus, in a sense, where natural intelligence decreases, it becomes more similar to artificial intelligence.

The last remark, when taken in the context of the Chinese room story, hints at a challenge to the partisans of strong Artificial Intelligence. In order to vindicate their claims, they should create entities able to simulate not only formal mode of reasoning but also all the other ones, that is instinctive, intuitive, and formalized. The last — let it be reminded — complies with the formal inference rules but has a semantic interpretation as well. But even if such a success is not very likely, the actual achievements of Artificial Intelligence prove very instructive from the rhetorical point of view.

What Leibniz imagined as blind thinking, what the champions of modern logic developed as the theory of formal proof and formal system, becomes physically materialized in the machines and programs devised in our times. And this is also a step towards developing a logic relevant to rhetorical purposes.