STUDIES IN LOGIC, GRAMMAR AND RHETORIC 11 (24) 2007

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COMPUTATIONAL DYNAMICS OF COMPLEX SYSTEMS
A NEW WAY OF DOING SCIENCE

A comment on *Thinking in Complexity. The Computational Dynamics of Matter, Mind and Mankind* by Klaus Mainzer, Springer-Verlag 2004 (4th Edition). Polish translation: *Poznawanie złożoności. Obliczeniowa dynamika materii, umysłu i ludzkości*, Wydawnictwo UMCS, Lublin 2007, translated by a team run by Marek Hetmański.

There is a good message for those Polish philosophers who realize the enormous philosophical import of *thinking in complexity*: the mentioned book by Klaus Mainzer has become more available to Polish readers owing to the translation done by a team of philosophers and translators at UMCS (Maria Curie Skłodowska University) in Lublin, Poland.[1]

To account for why I regard this message so good, let me recall the following. The way of perceiving the world which in the book is aptly called *thinking in complexity* is something like a creeping revolution both in science and in philosophy. It is a revolution, indeed, as it radically changes our world perspective. It is creeping as no single great event announced its start, and even no single branch of learning might be named as its proper terrain, hence a specialist in one field alone may overlook its emergence and significance.

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[1] At the start, let me hint at the intention of these comments. I do not intend to offer a paper in which the book under review gets examined for the correctnes of its statements and methods. This would require from the reviewer a level of expertise comparable with that found in the book, and this is not the case in question. Instead, I take the attitude of an appreciating reader who wishes to encourage fellow philosophers to make acquaintance with the book, as well as address the Author with certain issues. So my text distinguishes some items being attractive from a philosophical point of view, and puts some questions concerning philosophy of mind and epistemology. A special item might be devoted to the quality of Polish translation, but such an assessment should be made (and will, hopefully, be) in Polish.

ISBN 978–83–7431–***–* ISSN 0860-150X

The revolutionary initiative is due to mathematical logic at this point from which theoretical computer science (informatics) has emerged, that is, the discoveries concerning computability contributed by Gödel, Turing, Post and companions. This, so to speak, software complexity is deeply entangled with the complexity of hardware, the latter meaning dynamic systems changing in time as studied by physics, technology, biology etc.

Quite a number of Polish philosophers and mathematicians are familiar with the problems of computability, owing to contributions to this field as made by their natives Alfred Tarski, Andrzej Mostowski, Andrzej Grzegorczyk and others. That is to say, they are fairly familiar with the software side of the complexity. However, a knowledge of complex dynamic systems, and of how these are related to computability studies, is far from being eminent among our philosophers. Thus the translation of Professor Mainzer's book – to be, hopefully, duly disseminated in academic circles – should assist Polish scholars in their endeavours to keep in line with current science.

In what follows, (1) I introduce some key concepts in a way which should fit into the Polish philosophical audience's interests and conceptual equippment. Then, (2) I briefly survey the book content to let prospective readers know what they may expect from it. At last (3) I put some questions of how the study of complexity can profit from some promising research in automated theorem proving.

1. Some notions of complexity historically explained

1.1. Klaus Mainzer belongs to that circle of scholars who claim the rise of the new science of complexity. That circle includes Stephen Wolfram whose monumental book (2002) bears the much speaking title *A New Kind of Science* to resemble Galileo's phrase *nuove scienze* (in the title of his famous treatise of 1638). Such a claim alludes to the notions of *new paradigm* and *scientific revolution*, as introduced by Thomas Kuhn, which entered the vernacular of historians and philosophers of science. Hence it is reviewer's task to scrutinize to what extent such a far-reaching claim can be justified with reliable evidence. This involves an assessment of the essential point, opening the Author's Preface to the Fourh Edition; it runs as follows.

> The first edition of this book, published in 1994, began with the statement that the science of complexity would characterize the scientific development of the 21th century. In the first decade of this century, the prediction has been confirmed by overwhelming new empirical results and theoretical insights of the physical and biological sciences, cognitive and computer sciences, and the

> social and economic sciences. Complexity and nonlinearity are still prominent features in the evolution of matter, mind and human society. Thus, the science of complexity still aims at explanation for the emergence of order in nature and mind and in the economy and society by common principles.

This opening paragraph puts in a nutshell what the book is to tell. It should be read jointly with the book's title. In this title *Complexity*, as the category *in* which one should *think* about the world, is linked with *Computational Dynamics* of the universe, which is meant as involving *Matter, Mind, and Mankind.*

Before going deeper into the subject, it will be in order to address some conceptual troubles in our thinking about complexity.[2]

This is the very concept of 'complex system' as liable to be burdened with too many roles and a kind of redundancy. When speaking of a *system*, one thinks about an object having a number of elements which are in a way interrelated, hence a complex object. Then the phrase 'complex system' would mean 'complex complex object'. To avoid such a redundancy, let us distinguish the two following notions of complexity.

(1) The most general notion refers to any system *qua* system, that is, any set having elements interrelated with one another. A system may be either (A) abstract and static, as are the domains of mathematical theories, computer programs, etc, or (B) dynamic, that is, changing in time as are bodies (except, presumably, elementary particles), ecosystems, minds, societies, etc.

(2) A more specific notion refers only to dynamic systems (mentioned above in 1 as B), and – moreover - just those among them which behave in a way that we call nonlinear; this is a feature that makes a system complex in this more restricted sense.

Note, besides those listed in (2) there are dynamic systems that possess the feature of linearity, hence behave in a regular and predictable way, like our old good earth, also its companion the moon (this is why we happen to be so successful in predicting eclipses); those do not deserve to be called complex in this special sense. As being systems, they are complex in that

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[2] *'When I make a word do a lot of work like that', said Humpty Dumpty, 'I always pay it extra.'*. This was the replay to Alice's: *The question is whether you can make words mean so many different things.* (Lewis Carroll, *Through The Looking Glass*). This is exactly what happened to the term *complexity* in academic vernacular. It has got overworked with too many meanings, so there is a need to distinguish among them. A specially needed distinction is between the complexity of a dynamic systems due to its nonlinearity, and the complexity of a static system which depends on a number of elements and their interrelations. In the latter case (here under discussion) it is convenient to make use also of the comparative form, and so be allowed to talk about greater or smaller complexity.

most general sense mentioned in (1), hence we need a term to distinguish their type of complexity from that characteristic of nonlinear systems. For this purpose, we may devise the phrase *rudimentary complexity.*

In what follows, I am to use the single term *system* to refer to objects characterized by complexity in the most general sense as defined in (1) above, that is, embracing both rudimentary complexity and that possessed by nonlinear systems. Let the phrase *dynamic system* refer to any systems changing in time, and let the phrase *complex dynamic systems* denote those dynamic systems which are nonlinear.

1.2. Since I wish to encourage philosophers to take advantage of Mainzer's work let me start from their favourite phrase 'already the ancient Greeks' to mention two ancient insights, that of the Atomists and that of the Stoics.

Either of them contributed to the truth that the dynamics of the universe depends on two factors, to wit hardware and software. And each of them, when contributing one half of this truth to the picture, at the same time, ignored the other part. While the Atomists explained the universe in terms of hardware (atoms and space) alone, with Stoics the whole dynamics of the universe was explained by what might be compared to a software.

According to the Stoics, there is a ubiquitous system of non-physical units called in Greek *logoi spermatikoi*, in Latin (Augustinus) *rationes seminales*, what is being rendered in English as 'seeds of reason', 'germinating ideas', or else 'seminal plans'; the last phrase is the fittest for this discussion. While human-made things are produced according a plan devised outside these things themselves, to wit by a human mind, the things in Nature possess the designs of their construction and evolution inbuilt somehow inside. Such an idea might have come from observing seeds of plants, as suggested by the adjective 'seminal'. The tenets of either side can be summed up as follows.

(1) Atomists: the more complex a system is, that is, the more it involves elements and their interrelations as a physical object, the more problems it is able to solve.[3]

(2) Stoics: the ability of problem-solving depends on the kind and size of the seminal plan that is responsible for the development of the system in question, that is, the processes of solving its vital problems.

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[3] As to the problem difficulty, it may be measured with the number of partial problems, down to single lines of a program or a proof, whose solutions are steps in the way towards the target solution, as can be seen in proving theorems. Thus the degree of difficulty reduces to the multiplicity of problems.

Now let us compare either position with the following statements.

(1*) The more complex a processor – *hardware* – is, that is, the more it involves elements and their interrelations, the more problems it is able to solve (as can be seen in the example of high-scale integration devices).

(2*) The ability of problem-solving depends on the kind and size of program – *software* – that is responsible for the solving of problems by the system in question.

When comparing 1 with 1*, we realize that the Atomists grasped a rudimentary level of complexity, the same which is exemplified with such structures as electronic chips. What they did not succeed to grasp will be discussed a bit further.

When comparing 2 with 2*, we realize that the Stoics did not imagine any quantitative estimations of the power of plan relative to its complexity. This can be expressed by recalling that the great German pioneer of computer science Konrad Zuse introduced the concept of program under the German name *Plankalkül*. Thus we notice that the Stoics had an idea of plan, without having any idea of calculus. This sheds light on the giant distance between modern science and its philosophical anticipations.

From the Atomists up to the second half of the 20th century their rudimentary conception of complexity dominated both in philosophy and in science. Physical systems being complex in that manner, easily tractable with linear equations, were what Newtonian physics dealt with. A more refined concept appeared with inquiries into the role of feedbacks in dynamic systems; this led to paying special attention to non-linear dependencies. Let the link between positive feedback loops and the phenomenon of non-linearity be explained with the following example. It exemplifies how some unpredictable processes emerge in systems which previously behaved in a predictable way.[4]

> Imagine a microphone which induces a loud squeal from a speaker when the microphone gets too close to the speaker. The positive feedback occurs because the sound picked up from the microphone is amplified, sent out through the speaker and returns to the microphone to be picked up louder than before. Now imagine a system consisting of many microphones randomly connected with wires, as well as many speakers. The probability of the emergence of powerful feedbacks increases as more elements are added, more interconnections are

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[4] The example is taken from the text *Multicellular Computing: Dynamic Complexity*, see: evolutionofcomputing.org/Multicellular/DynamicComplexity.html.

> added, or the elements themselves become more complex and therefore can interact with others in more complex ways. Thus, any change to the system that increases the number of possible feedback loops increases the probability of such an emergent phenomenon.

This is a nice example since it exemplifies both feedback loops and transition from linear to non-linear process. Let us note, there is an interval of distances between a speaker and his microphone in which the squeal does not appear, and thus there holds a linear dependence between the distance in question and the sound power: the closer the microphone, the more intense is the sound, but without any unexpected events – up to a certain point beyond which a squeal emerges, and gets more and more intense; thus the process, so far having been linear, starts to be non-linear.

2. A survey of contents

2.1.[5] The introductory chapter emphasises the novelty of the theory of nonlinear complex systems as well as its successes in problem solving in natural and in social sciences. The novelty consists in discovering and explaining the feature of 'emergence of certain macroscopic phenomena via nonlinear interactions of microscopic element in complex systems'. This new approach opposes the paradigm of reductionism which the Author exemplifies, with regard to mental and social phenomena, by mechanistic explanation as offered by Hobbes, Lamettrie etc.

According to the traditional paradigm of science, including the Newtonian mechanics, our macroscopic world would be – as a rule – linear, while nonlinearities would be negligible exceptions. Contrary to that view, recently we start to realize (what Poincare anticipated a century ago) that as a rule we deal with nonlinear systems, while linear ones are exceptional. In the macroscopic world of quanta, in spite of certain aspects of linearity, the quantum world is not linear in general. Thus, the picture of the whole science as seen by the Author is like Stephen Wolfram claim (see 1.1 above) that we enter into the age of new science.

2.2. In the next chapter we encounter a feature of the book, characteristic of the next chapters too, namely a combine of philosophical interpretations, merged in the history of philosophy, with technical discussions involving

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[5] In the numbering in this section, each second digit corresponds to the so numbered chapter in the book.

a considerable knowledge from various fields of science: not only Newtonian physics but also quantum physics, thermodynamics, chemistry, biology, economics etc., supported by a necessary piece of mathematical apparatus. This is a reason, indeed, to appreciate the Author's expertize, and at the same time to caution philosophers, who as a rule hardly share such a competence, that the reading may appear a bit stressing. However, this by no means should discourage prospective readers, since a substantial lot of knowledge and understanding can be obtained from the book as a whole, in spite of local difficulties.

This chapter, entitled 'Complex Systems and the Evolution of Matter' starts from a historical background, dealing first with philosophical anticipatory insights of Aristotle and Heraclitus concerning the question: 'how can order arise from complex, irregular, and chaotic states of matter?' Another historical survey outlines the picture of deterministic and linear universe as found in Newton, Einstein and Laplace. The next sections reveal various departures from determinism and linearity.

A section most attractive from a philosophical point of view deals with the question of the emergence of order in cosmic evolution. No definitive answer is available at the current stage of research, but we learn from this section about several alternative models which were considered, as Hoyle's stationary universe, Linde's idea that our universe is involved in a fractal multiverse (a set of universes), and the string theory – an attempt to unify the four basic kinds of interactions as generated by oscillating strings. The last alternative is optimistic, giving a chance to avoid the loss of information (as stored in the strings) in black holes.

2.3. In the Chapter entitled 'Complex Systems and the Evolution of Life' we find illuminating remarks concerning the first integration of the idea of information, physics (thermodynamics) and biology (theory of evolution) elaborated by Boltzmann (1844-1906). It should be of special interest for philosophers as those who look for a synthetic picture of the world. Boltzmann's synthesis anticipated the very foundations of modern scientific philosophy.

Highly ordered complex systems, such as plants and animals, are most highly improbable forms in the light of the second law of thermodynamics. This law says that entropy (to be roughly equated with disorder and minimal information) grows with time, hence the intelligent life, as being an apex of order, should not have arisen in our universe. In the story in which this paradox gets explained, it is the sun which plays the role of the main hero. From it the earth receives energy to compensate the spontaneous increase of entropy in organisms. This is a crucial idea to elucidate the origins of life and its further evolution.

This idea sheds light on a universal feature of life and intelligence. There may be surprisingly many forms of life in the universe, many of them lacking any similarity to our familiar earth life, but no one can escape this dependency on the input of energy. Thus Boltzman's ideas have contributed to the philosophically fascinating issue of how energy and information are related to each other.

2.4. The chapter 'Complex Systems and the Evolution of Brain' deals with the emergence of brain and mind. Let me focus on one chosen point to recommend it for philosophical reflexion. It is found in the following passage concerning the neuronal basis generating consciousness (p. 172).

> Concerning consciousness [...], it is assumed that global activation of cell population, as exerted by the recticular formation on the cortex, would generally increase the probability of assemblies being formed. [...] The production rate of cell assemblies determines the amount, complexity, and duration of representations of sensory patterns from the outer world, for instance. *Consciousness is a self-referential state of self-reflexion. Thus, a conscious state is based on a cell assembly representing an internal state (and not only a state of the outer world.* [Italics – WMF] For example, I not only have the impression of a green tree, but I am conscious that I am looking at this tree.

Two comments will be in order, one to hint at the weakness of behaviourism, and one to exemplify the nonlinear dynamics and emergence in mental processes. Some examples in these comments are not taken from the chapter reported; however, they are meant to shed light on certain points in it that deserve special attention from a philosophical point of view.

According to behaviourists, getting rid of the concept of consciousness is necessary for any progress in psychology – for two reasons. First, the concept of consciousness is empty, it is a product of metaphysicians' phantasy. Second, even if fictional entities might sometimes be of use in science, this one proves of no use. The job of psychology consists in registering how certain stimuli are invariably associated by observable reactions. Behaviourists happen to refuse studying the nervous system, they remain satisfied with the results of their experiments corelating no more than stimuli and reactions.

This was a dogmatic attitude ignoring facts and losig chances to understand actual interactions. Among the ignored facts there is that mentioned in the quotation given above (the point italicized). Let X be an assembly of cells mapping a state of the outer world (e.g., the redness of a rose), and $M(X)$ be the state of X which consists in the mapping in question. Then there is another assembly of cells, say Y, to map the internal state $M(X)$, and this internal state of Y may be mapped by another internal state (one of a higher order, so to speak), and so on.

Some processes of development of intelligence consist in positive feedbacks between cell assemblies as those described above. When dynamic elements in a system, like those named above as X and Y, interact with other dynamic elements, they generate positive feedback loops. Then qualitatively much different and surprising new phenomena may emerge.

Analogous interactions, let me add here, occur in social processes of the evolution of intelligence. Consider, for instance, the development of logic from Socrates' teachings, twined with the progress of mathematics, up to the current theory of computability as a basis of computer science. I refer to that historical process as being well-known from numerous sources, but something like that appears in joint processes of learning logic, mathematics and computer science by an individual mind/brain.

Socrates dialogues are illuminating as a practice of reasoning combined with his ad hoc comments regarding correctness of the reasoning in question. In such comments there arises a primary logical consciousness. Suppose that Plato, owing to these beginnings of logical theory, improved such a practical art of reasoning so, that Aristotle had at his disposal a rich repertoire of forms of reasoning practised in Academia, and then he reflected upon them in a systematic way. Thus the first systematic logical theory has been born.

About the same time Euclid erected a wonderful edifice – a parennial paradigm of axiomatic mathematical practice. For centuries it provided logicians with an abundant material for logical reflexion. Owing to both streams of reflexion, that on Aristotle's theory and that on mathematical practice (e.g. a refined use of quantifiers in Calculus), mathematical logic emerged with Boole and Frege. This helped to improve mathematical practice, as methods of investigating consistency, completeness, decidability etc. Problems of decidability, in turn, gave rise to the study of computability, as initiated by Gödel and Turing, and that blossomed with the idea of computer from which a new period of civilization begins. This is, indeed, an emergence of something surprisingly new, a new civilization, from a cycle of positive feedbacks involving at the start the Greek passion for arguing and the ancient mathematical practice, both events fairly remote from the final result.

Such a historical process happens to be mapped by an individual development of an individual philosophical mind. His education should include an intense practice of reasoning which reinforces and gets reinforced by a theory of logic. It should include too a portion of mathematics to supply a material for logical reflexion, which in turn deepens the understanding of mathematics, science and cognition.

The moral to be drawn from such stories is to the effect that emergent phenomena are due to strings of positive feedbacks, and so result from a nonlinear growth. This is also a practical receipe for success in business, national economy, cultural development, etc. One has to deliberately employ such factors that they will interact in the positive feedback mode. Then emerge results which would astonish us as miracles, were we not conscious of their being natural in the nonlinear world of brain/mind and society.

2.5. From a philosophical perspective (outlined below in Section 3), the chapter 'Complex Systems and the Evolution of Computability' is central to this book. Its opening paragraph runs as follows.

> (1) *The evolution of complexity in nature and society can be understood as the evolution of computational systems.* In the beginning of modern times, Leibniz already had the idea that the hierarchy of natural systems from stones and plants up to animals and humans corresponded to natural automata with increasing degrees of complexity. (2) The present theory of computability enables us to distinguish complexity classes of problems, meaning the order of corresponding functions describing the computational time of their algorithms or computational programs. But we can also consider the size of a computer program when defining the algorithmic complexity of symbolic patterns. [Numbering and emphasis – WM.]

The chapter consists of four sections whose contents can be put in a nutshell as follows. Point (1) is discussed in the first section of the chapter, point (2) in the second section. The third section deals with information entropy in complex systems; since the approach in terms of such information dynamics does not account for human knowledge processing, it is asked whether a higher efficiency of information processing might be brought with quantum computing. The fourth (last) section, after recalling Leibniz's idea of automaton, presents John von Neumann's cellular automata (CA) framework as capable to account for chaos and randomness in complex systems.

In section 1 the first sentence (italicized in the quotation above) may figure as a motto for the whole book as it explains the key phrase *computational dynamics* in the title of the book. The phrase is to mean that the dynamics of the universe (i.e., matter, mind and mankind) possesses the essential feature of being *computational*.

What should it mean? Let me suggest that this dynamics consists in a rapid increase of computational power of the universe as a processing information device – somehow like devices referred to by the famous Moore's Law. In fact, the computational power of the universe does dramatically expand with the rise of life, then the appearance of brain, then of society, civilization, esp. information civilization, and so on. It is in order here to refer to what Barrow and Tippler endorse as *Final Anthropic Principle.*[6]

> *Intelligent information-processing must come into existence in the Universe, and, once it comes into existence, it will never die out.*

Even if one does not share the optimistic 'will never die out' (a view connected with a specific approach of the authors), the picture of such a dynamic evolution of information-processing (that is, computing) activity may us help in realizing a cosmological content of the notion 'computational dynamics'.

To contribute to the discussion of this chapter, let me sketch the question of limits of computational power. The limits of the Universal Turing Machine are well-defined by Church-Turing Thesis. Now, the crucial question is the following: are they the same for the human mind? Klaus Mainzer is cautious in proposing a definite answer, but an inclination of him may be read from his summary of the chapter in question as found in the introducing chapter. He puts a question in which it is assumed for granted that there are limitations to the analogies of computers with human mind and brain. The question based on this assumption is to the effect: do these limitations result from Gödel's and Turing's results of incompleteness and undecidability? No direct answer is found in this passage but an indirect one may be inferred from what the Author says of Stephen Wolfram results concerning CA. He refers to the result that all kinds of nonlinear dynamics can be simulated by CA.

For a final conclusion two more premisses are needed, one of them being already known; as to the second, it can be reasonably guessed to be held by the Author. The known one is to the effect that CA are equivalent to universal Turing machines (hence limited by Church-Turing Thesis), though in practice much more efficient. The premiss to be guessed would say that the nonlinear activity of the brain, that gives it so great advantages ower electronic computers, is just of the kind studied with Wolfram's research on CA. Now, since brains are CA, and those are equivalent to universal Turing machines, it would follow that brains are equivalent to universal Turing

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[6] John D. Barrow and Frank J. Tipler, *The Anthropic Cosmological Principle*, Oxford University Press, 1996. See p. 23.

machines; and, since they are like CA, they share their enormous efficiency with those von Neumann's creatures.

This is a hypothetical line of reasoning whose conclusion I am to use to put the following question. Let us assume (as I guess to be assumed by the Author) that our brains/minds are CA: does this assumption accounts for the phenomenon of mathematical creativity? Emil Post who designed an abstract computing machine, which has proved equivalent to the universal Turing machine, firmly refused to believe that such a machine could simulate human creativity in proving theorems. Here is one of his numerous utterances in this question.[7]

> The logical process is essentially creative. This conclusion [...] makes of the mathematician much more than a kind of clever being who can do quickly what a machine would do ultimately. We see that a machine would never give a complete logic; for once the machine is made we would prove a theorem it does not prove.

In Section 3 I am to tell about some experiences which shed light on the phenomenon of creativity, as referred to by Post, and on its relation to algorithmic complexity. Since the point is crucial from the perspective here adopted, let it be the last item of this review. However, this has to be at the cost of giving up a discussion of the three ending chapters: the 6th concerning complexity in the evolution of artificial life and artificial intelligence, the 7th concerning complexity of evolutionary social and economic processes, and the 8th dealing with some philosohical issues. They offer such a reach supply of facts, ideas and problems that it will be more advisable to handle them in a special paper which I feel likely to write in a due time.

3. Insights producing algorithms to reduce complexity Are such insights themselves produced by algorithms? Exemplification through the 'Curious Inference' story

3.1. The question in the title above could be restated in the form: is the dynamics of mind in a full manner computational? The latter form refers to the *computational dynamics of mind* as called in the title of the book discussed. Should it be answered in the affirmative, then the whole dynamics of mind,

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[7] Emil Post, *Absolutely Unsolvable Problems and Relatively Undecidable Propositions – An Account of an Anticipation* in: *Solvability, Provability, Definability. The Collected Works of Emil L. Post*, edited by Martin Davis, Birkhäuser, Boston (etc.) 1994. See p. 428, footnote 101.

not only when using algorithms, but also when eliciting insights, would entirely result from some algorithms – recorded, presumably, in a brain code.

Suppose, there exists a hard problem, that is such that no algorithmic (mechanical) intelligence is able to handle it, while it gets solved efficiently with an insight of human intelligence.

Before I discuss such a situation, let me suggest some handy terminological devices. Let the algorithmic intelligence be called a *robot*, and the insightful one – a *daemon* (in Unix slang, the name to mean a process runnig under its own account, as 'behind the scene').[8] In the enterprise of proving theorems, especially in mathematics, robots produce formalized proofs, i.e., such that their checking for logical correctness is carried strictly according to an algorithm. Let us call them *algorithmic proofs*, and those which require an insight let be called *intuitive proofs*. Now robots can be defined as entities whose proving ability is restricted to algorithmic proofs while daemons are capable of intuitive proving as well.

The story named in the last line of this section's title tells about experimenting with a problem which so far has proved too hard for any robot (i.e., any prover system), and easily solvable for a human daemon. The story goes back as far as 1936, when Kurt Gödel published a short comment regarding the complexity of algorithmic (formalized) proofs. Such a complexity is identified either with the number of steps (operations) in the course of proving or with the number of symbols occurring in a proof; the latter is the case here. It is crucial that the proof be formalized (within a system of formal logic), since only then no symbols are likely to be omitted (while in intuitive proofs some steps are omitted, if obvious for a supposed class of daemons). Owing to such, so to speak, typographical completeness, the number of symbols can be adopted as a measure of complexity. The paper offered by Gödel bears the title *Über die Länge von Beweisen* – On the Length of Proofs, where the length means a number of symbols, that is, the kind of complexity which in the moment we have in mind.[9]

Gödel discovered the phenomenon which nowadays is called *speedup* in the efficiency of proving; let me call it the *Gödelian speedup*. The respective Gödel's statement (English translation) runs as follows (italics – WM).

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[8] More discussion on Unix daemon as a model of intuitive processes is offered in my paper *Rational Belief as Produced by Computational Processes* in: *Foundations of Science*, vol. 2, no. 1, 1997, Kluwer Academic Publishers.

[9] Kurt Gödel, *Über die Länge der Beweisen* in: *Ergebnisse eines mathematischen Kolloquiums* Heft 7, Franz Deuticke, Leipzig und Wien 1936. From among three forms of this term, 'speed up', 'speed-up' and 'speedup', I choose the last which is reported in Webster (though in a different meaning, occurring in economics, see www.merriam-webster.com/dictionary/), and also in some texts concerning data processing.

> Thus, passing to the logic of the next higher order has the effect, not only of making provable certain propositions that were not provable before, but also of *making it possible to shorten, by an extraordinary amount, infinitely many of the proofs already available.*

The first part of this statement says what is involved in Gödel's groundbreaking paper of 1931, while the second (italicized) tells something new, which is of great practical import for computer science. Let us dwell a while on the method of reducing complexity as suggested by that statement. Its significance lies in the fact that it opens the following questions.

- A. How great is that extraordinary amount by which a proof gets shortened, i.e., its complexity gets reduced, owing to the passing to the next higher order? How much is such a reduction significant practically?
- B. Is this method (of complexity reducing) (B1) obtainable in the same degree for human provers and mechanical provers (the latter dealing solely with algorithmic proofs)? If not the same, then (B2) how big may there be such a difference of degree, and (B3) how should it be accounted for?
- C. Provided its availability for mechanical provers, how great would be the difference when comparing complexities of the shortest proofs of the same theorem, one produced by a human prover (daemon) and one by a mechanical prover (robot)?

3.2. Question A has been answered after a half century hiatus by George Boolos in his seminal paper of 1987, attractively titled *A Curious Inference* (from now on cited as BP – for Boolos' Proof), which offered an enlightening case study. I can feel free from reporting wider on BP, since its contents is summed up in a preceding, easily available, volume of this journal's electronic version.[10] Let me just mention what counts most: that Boolos' proof of an arithmetical theorem (concerning a property of Ackermann function) when performed in the second-order logic takes one page alone. On the other hand, the number of symbols occurring in the first-order derivation is represented by the exponential stack of as many 2's as 64536, that is, larger than any integer that might appear in science. This is a dramatic result, indeed, which nicely exemplifies the Gödelian speedup.

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[10] See logika.uwb.edu.pl/studies/vol9.htm. Roman Murawski, *The Present State of Mechanized Deduction, and the Present Knowledge of its Limitations* in: Witold Marciszewski (Ed.), *Issues of Decidability and Tractability*, vol. 9 (22), 2006 of the journal *Studies in Logic, Grammar and Rhetoric.* A sketchy discussion on this subject is also found in the same volume, in the paper by Witold Marciszewski: *The Gödelian Speed-up and Other Strategies to Address Decidability and Tractability.*

The same result answers the second part of Question A. Since a proof in the language of first-order logic is so much intractable, the adopted method of reducing the proof to such a short, so easily tractable, version is of enormous practical import. At the same time, Boolos to some extent paved the way to answering question B2 as discussed below.

The next two decades after BP have brought forth some answers to the remaining questions, due to an intense research in automated theorem-proving. As for B1, the answer is decidedly in the negative. In point B2 it is continued to the effect that differences of degree are colossal, and the source of them (asked in B3) lies in the fact that a great deal of creative intuition is necessary, which so far is a priviledge of humans, unattainable to robots. Both Boolos and automated reasoning researchers refer to the set-theoretical axiom of comprehension, essential for the proof under study, as one whose applications much require creative insights. This axiom (equivalent to a formula in the second-order language) makes it possible to introduce new concepts needed for the proof in question.

To win more understanding, let me give a voice to experts in automated reasoning research. The most illuminating guide to our problem I could find so far is the survey by Benzmüller and Kerber [2001] entitled *A Challenge for Mechanized Deduction.*[11] In the report on their research they conclude as follows.

> Boolos' example perspicuously demonstrates the limitations of current first-order and higher-order theorem proving technology. With current technology it is not possible to find his proof automatically, even worse, automation seems very far out of reach. Let's first give a high-level description why this is so. Firstly, Boolos' proofs need comprehension principles to be available and it employs different complex instances of them. [...] Secondly, the particular instances of the comprehension axioms cannot be determined by higher-order unification but are so-called Heurika-steps which have to be guessed. However, the required instantiations here are so complex that it is unrealistic to assume that they can be guessed. [...] Here it is where human intuition and creativity comes into play, and *the question arises how this kind of creativity can be realised and mirrored in a theorem prover.* [Emphasis – WM]

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[11] See Christoph Benzmüller and Manfred Kerber, *A Challenge for Mechanized Deduction*, 2001 (www.cs.bham.ac.uk/ mmk/papers/01-IJCAR.html). A considerable number of other studies on this subject can be found with the search: citeseer.ist.psu.edu/. There may be of special interest the paper by Natarajan Shanker *Using Decision Procedures with a Higher-Order Logic* which refers to excellent surveys of higher-order logics as offered by S. Feferman, J. van Benthem, etc.

3.3. As for question C (regarding complexity differences between algorithmic and intuitive proofs), its relevance can be noticed with the following consideration. What may be a tractable size of a mathematical proof carried by a human mathematician? A highly interesting evidence is due to the fact that the famous proof of Fermat's great theorem by Andrew Wiles (published in *Annals of Mathematics*, May 1995) required 200 manuscript pages, that is, approximately, 400000 bits (symbols), and must have been divided into six parts to become readable for each of the six reviewers asked by the editor to critically read and comment on the proof. It is no formalized proof, hence it must have gaps to insightfuly be filled up by a competent reader.

Certainly, after filling up such gaps according to the rigours of algorithmic methods, the proof has to get longer. How much longer? Would have such a difference a practical relevance concerning time and memory size for data processing? Unfortunately, we have no answer in this individual instance, but we can consider a case having already been subject to a study, and so obtain instructive analogies. Such is the research on automated proof checking reported by Christoph E. Benzmüller (the University of Cambridge and Universität des Saarlandes) collaborating with Chad E. Brown (Universität des Saarlandes).[12] The authors possess a unique expertise in comparing *automatic proving* (which they call full automation) with *automatic proof-checking* as performed by systems like Mizar and OMEGA. As for automatic proving, they conclude, thus confirming the conclusion of the previously quoted paper (by Benzmüller and Kerber, 2001), with the following statement.

> The full automation of Boolos curious inference seems not to be in reach and it will be a challenge problem to automated theorem proving for a long time to come.

However, owing to the procedure of the automated checking (for logical correctness) of Boolos' inference, we can fancy a minimum size (hence a minimum complexity) of the same proof if produced by a robot (here, a system being an automatic prover). Let us suppose that a proof written by a human in a language specially devised for proof-checking (as Mizar, OMEGA, etc.)

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[12] Christoph E. Benzmüller and Chad E. Brown, *The Curious Inference of Boolos in Mizar and OMEGA* in: Roman Matuszewski and Anna Zalewska (Eds.), *From Insight to Proof. Festschrift in Honour of Andrzej Trybulec*, vol. 10 (23) of the journal *Studies in Logic, Grammar and Rhetoric*, University of Białystok, Białystok (Poland) 2007. See also: logika.uwb.edu.pl/studies/. A. Trybulec, let me add, is the author of the system Mizar referred to in the mentioned title.

will be roughly of like size as the proof of the same theorem produced by an automatic prover. Thus we obtain (according to the paper by Benzmüller and Brown) the following sizes of compressed (using gzip) files.

— Boolos' proof sketch: 637 bytes;
— Mizar article: 2310 bytes;
— OMEGA article (in one of the versions produced): 2602 bytes.

Obviously, an article being a proof (of the same theorem) which would be produced by an automatic prover might considerably differ in size from those mentioned above. However, we can reasonably estimate that the order of quantity would be preserved.

Thus we can realize that the carrying of a proof in the second-order language dramatically lowers its complexity (as compared with first-order procedures) in any case; that is, independently of technology adopted, be it the old-fashioned pencil-and-paper technology, be it the modern computer technology.

What makes the problem hard for computer is that its solution requires a creative insight which so far is not obtainable for robots. This brings the problem of choosing between two philosophical options which are as follows. (α) Is there so that insights belong to a category of cognitive acts entirely disjoint with the class of algorithms? Or, rather, (β) any insight is due to a hidden algorithm? (Hidden in the sense that we cannot recognize this fact for limitations of our current knowledge.)

If the latter is the case, then a brain as that of Boolos should contain a second-order algorithm somehow recorded in a brain code (being like machine code in a computer). As being algorithmic, it does not exceed capabilities of a robot, to wit a universal Turing machine. Then no daemon is needed, to do the job of, for instance, inventive using the axiom of comprehension.

This statement of options leads to a final comment on the book under discussion. It takes the form of the following question. The title of the book contains the term *the Computational Dynamics of Mind* ('computational' amounts to 'algorithmic'). Should it mean that the whole dynamics of mind is computational? Or, that there is a kind of mental dynamics which is computational, while another one is not computational, and it is the former which belongs to the subject matter of the book (the latter being outside the scope of the Author's interests)?

The remaining items, Matter and Society (Mankind) might be addressed with an analogous disjunction. However, this would require a special discussion, at least as detailed as that devoted here to the computability of mental dynamics. Let it be left to another opportunity.